1st Year Mathematics

Great Science Academy Guess Paper By

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For All Punjab Boards (LHR, GRW, SGD, SWL, MTN, RWP, DGK, FSD & BWP)

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Chapter No.01

Number System

- ## Short Questions

1. Definitions

(i)	Rational Number	(ii)	Irrational Number	(iii)	Terminating Decimal
(iv)	Recurring Decimal	(v)	Non – terminating	(vi)	Non – recurring
(vii)	Addition Law	(viii)	Multiplication Law	(ix)	Properties of Equality
(x)	Complex Number	(xi)	Imaginary Number	(xii)	Modulus C.N

2. Prove that $\sqrt{2}$ & $\sqrt{3}$ is an irrational numbers
3. State the closure property w. r. t. addition and multiplication
4. Prove that
   - (i) $\frac{a}{b}=\frac{c}{d} \Leftrightarrow ad=bc$ **(Principle for equality of fraction)**
   - (ii) $\frac{a}{b}.\frac{c}{d}=\frac{ac}{bd}$ **(Rule for Product of Fraction)**
   - (iii) $\frac{a}{b}=\frac{ka}{kb}$ **(Golden Rule of Fraction)**
   - (iv) $\frac{a/b}{c/d}=\frac{ad}{bc}$ **(Rule for Quotient of Fraction)**
5. Does the set $\{1,-1\}$ possess closure property with respect to
   - (i) Addition (ii) Multiplication?
6. Which of the following set have closure property w. r. t. addition and multiplication?
   - (i) $\{0\}$ (ii) $\{1\}$ (iii) $\{0,-1\}$ (iv) $\{1,-1\}$
7. Name the properties used in the following inequalities
   - (i) $-3<-2 \Rightarrow 0<1$ (ii) $-5<-4 \Rightarrow 20>16$
   - (iii) $a>b \Rightarrow \frac{1}{a}<\frac{1}{b}$ (iv) $a>b \Rightarrow -a<-b$
8. Prove that the following rules of addition
   - (i) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ (ii) $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$
9. Prove that $-\frac{7}{12}-\frac{5}{18}=\frac{-21-10}{36}$
10. Simplify by justifying each steps
    - (i) $\frac{4+16x}{4}$ (ii) $\frac{1/4+1/5}{1/4-1/5}$
    - (iii) $\frac{a/b+c/d}{a/b-c/d}$ (iv) $\frac{1/a-1/b}{1-1/a.1/b}$
11. Write any two properties of the fundamental operation on the complex numbers
12. Simplify the following
    - (i) i^9 (ii) i^{14} (iii) $(-i)^{19}$ (iv) $(-1)^{-21/2}$

13. Simplify
(i) $(7,9)+(3,-5)$ (ii) $(8,-5)-(-7,4)$ (iii) $(5,-4)(-3,-2)$ (iv) $(5,-4)\div(-3,-8)$

14. Prove that the sum as well as the product of any two conjugate complex numbers is real number

15. Find the multiplicative inverse of each of the following numbers
(i) $(-4,7)$ (ii) $(\sqrt{2},-\sqrt{5})$ (iii) $(1,0)$

16. Factorize the following
(i) a^2+4b^2 (ii) $9a^2+16b^2$ (iii) $3x^2+3y^2$

17. Separate into real and imaginary parts
(i) $2-7i/_{4+5i}$ (ii) $(-2+3i)^2/_{1+i}$ (iii) $i/_{1+i}$

18. Find moduli of the following complex numbers
(i) $1-i\sqrt{3}$ (ii) 3 (iii) $-5i$ (iv) $3+4i$

19. Prove that $\forall\, z_1, z_2 \in C$
(i) $|-z|=|z|=|\bar{z}|=|-\bar{z}|$ (ii) $z\bar{z}=|z|^2$
(iii) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ (iv) $|z_1.z_2|=|z_1|.|z_2|$
(v) $\bar{\bar{z}}=z$

20. If $z_1=2+i, z_2=3-2i$ & $z_3=1+3i$ then express $\frac{\overline{z_1}.\overline{z_3}}{z_2}$ in form of $a+ib$

21. Show that $\forall\, z_1, z_2 \in C, \overline{z_1.z_2}=\overline{z_1}.\overline{z_2}$

22. Find out real and imaginary parts of each of the following complex numbers
(i) $(\sqrt{3}+i)^3$ (ii) $\left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^5$

23. Find the multiplicative inverse of each of the following numbers
(i) $-3i$ (ii) $1-2i$ (iii) $-3-5i$ (iv) $(1, 2)$

24. Simplify
(i) i^{101} (ii) $(-ai)^4, a\in R$ (iii) i^{-3} (iv) i^{-10}

25. Prove that $\bar{z}=z$ iff z is real

26. Simplify by expressing in the form $a+ib$
(i) $5+2\sqrt{-4}$ (ii) $(2+\sqrt{-3})(3+\sqrt{-3})$ (iii) $\frac{3}{\sqrt{6}-\sqrt{-12}}$

27. Prove that $\forall\, z \in C$
(i) $z^2+\bar{z}^2$ is real number (ii) $(z-\bar{z})^2$ is real number

28. Simplify
(i) $\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)^3$ (ii) $(a-ib)^3$ (iii) $(a+ib)^{-2}$ (iv) $\left(3-\sqrt{-4}\right)^{-3}$

Chapter No.02

Sets, Functions and Graph

• Short Questions

1. Definitions

(i)	Equal Set	(ii)	Equivalent Set	(iii)	Order of Set
(iv)	Proper Sub Set	(v)	Improper Sub Set	(vi)	Power Set
(vii)	Union of Two Sets	(viii)	Intersection of Two sets	(ix)	Complement of a Set
(x)	Proposition	(xi)	Tautology	(xii)	Absurdity
(xiii)	Quantifiers	(xiv)	Disjunction	(xv)	Implication
(xvi)	Conjunction	(xvii)	Function	(xviii)	Into Function
(xix)	Surjective	(xx)	Injective	(xxi)	Bijective
(xxii)	Binary Operation	(xxiii)	Groupoid	(xxiv)	Semi Group
(xxv)	Monoid	(xxvi)	Group	(xxvii)	Finite & Infinite Group

2. Write the following sets in set builder notation
(i) $\{0.\pm1,\pm2,\pm3...,\pm1000\}$ (ii) $\{-100,-101,-102\,...,-500\}$

(iii) {Peshawar, Lahore, karachi, Quetta} (iv) {January, June, July}
(v) The set of all real numbers between 1 & 2 (vi) The set of integers between – 1000 & 1000

3. Write each of the following sets in descriptive and tabular form
(i) $\{x|x \in N \wedge x \leq 10\}$ (ii) $\{x|x \in N \wedge 4 < x < 12\}$
(iii) $\{x|x \in Z \wedge -5 < x < 5\}$ (iv) $\{x|x \in P \wedge x < 12\}$
(v) $\{x|x \in O \wedge 3 < x < 12\}$ (vi) $\{x|x \in O \wedge 5 \leq x \leq 7\}$
(vii) $\{x|x \in O \wedge 5 \leq x < 7\}$ (viii) $\{x|x \in Q \wedge x^2 = 2\}$
(ix) $\{x|x \in R \wedge x = x\}$ (x) $\{x|x \in R \wedge x \notin Q\}$

4. Write two proper subsets of each of the following sets
(i) $\{a, b, c,\}$ (ii) N (iii) R
(iv) $\{0,1\}$ (v) W (vi) $\{x|x \in Q \wedge 0 < x \leq 2\}$

5. Is there any set which has no proper sub set? If so name that set.

6. What is the difference between $\{a, b\}$ & $\{\{a, b\}\}$?

7. Write down the power set of each of the following sets
(i) $\{a, b, c, d\}$ **(Example)** (ii) $\{+, -, \times, \div\}$ (iii) $\{a, \{b, c\}\}$

8. If $A = \{1,2,3,4,5\}, B = \{4,5,6,7,8,10\}$, then find A – B & B – A

9. Show that A – B and B – A by Venn diagram when
(i) $A \sqsubseteq B$ (ii) $B \sqsubseteq A$ (iii) A & B are overlapping sets

10. Let $U = \{1,2,3,4,5,6,7,8,9,10\}, A = \{2,4,6,8,10\}, B = \{1,2,3,4,5\}$ & $C = \{1,3,5,7,9\}$ List the members of each of the following
(i) A^c (ii) B^c (iii) A – B (iv) U^c
(v) $A \cup B$ (vi) $A \cap C$ (vii) $A^c \cup C^c$ (viii) $A^c \cup C$

11. Using Venn diagram to verify that the following
(i) $A - B = A \cap B^c$ (ii) $(A - B)^c \cap B = B$

12. State the De – Morgan's Laws

13. Write any two Properties of Intersection and Union

14. Verify the commutative properties of union and in intersection for the following pairs of sets
(i) $A = \{1,2,3,4,5\}, B = \{4,6,8,10\}$ (ii) N and Z

15. Verify the properties for the sets A,B & C given below
(i) Associativity of Union
(ii) Associativity of intersection
(iii) Distributive property of union over intersection
(iv) Distributive property of intersection over union
(a) $A = \{1.2.3.4\}, B = \{3,4,5,6,7,8\}$ & $C = \{5,6,7,8,9,10\}$
(b) $A = \{\ \}, B = \{0\}$ & $C\ \{0,1,2\}$
(c) N, Z, Q

16. Verify De – Morgan's Laws for the following sets
$U = \{1,2,3, \ldots 20\}, A = \{2,4,6, \ldots 20\}$ & $B = \{1,3,5, \ldots ,19\}$

17. Let U = the set of English alphabet, A = {x | x is a vowel} & B = {y | y is a consonant} verify De – Morgan's Laws for these sets

18. If $U = \{1,2,3,4,5 \ldots, 20\}$ & $A = \{1,3,5, \ldots 19\}$, verify the following
(i) $A \cup A' = U$ (ii) $A \cap U = A$ (iii) $A \cap A' = \varphi$

19. Taking any sets, say $A = \{1,2,3,4,5\}$ verify the following
(i) $A \cup \varphi = A$ (ii) $A \cup A = A$ (iii) $A \cap A = A$

20. Form suitable properties of union and intersection deduce the following results
(i) $A \cap (A \cup B) = A \cup (A \cap B)$ (ii) $A \cup (A \cap B) = A \cap (A \cup B)$

21. Prove that in any universe the empty set φ is sub set of any set A **(Example)**

22. Construct the truth table of $[(p \longrightarrow q \wedge p) \longrightarrow q]$**(Example)**

23. Write the converse, inverse and contrapositive of the following conditions
(i) $\sim p \longrightarrow q$ (ii) $q \longrightarrow p$ (iii) $\sim p \longrightarrow \sim q$ (iv) $\sim q \longrightarrow \sim p$

24. Construct the truth table for the following statements
(i) $(p \rightarrow \sim q) \vee (p \rightarrow q)$ (ii) $(p \wedge \sim p) \longrightarrow q$ (iii) $\sim(p \longrightarrow q) \leftrightarrow (p \wedge \sim q)$

25. Show that each of the following statements is a tautology

(i) $p \to (p \vee q)$ **(ii)** $p \longrightarrow (p \vee q)$ **(iii)** $\sim(p \longrightarrow q) \to q$ **(iv)** $\sim q \wedge (p \to q) \to \sim p$

26. Determine whether each of the following is a tautology, a contingency or an absurdity

(i) $p \wedge \sim p$ **(ii)** $p \to (q \longrightarrow p)$ **(iii)** $q \vee (\sim q \vee p)$

27. Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

28. Convert the following theorems into logical form

(i) $(A \cap B)' = A' \cup B'$ **(ii)** $(A \cup B) \cup C = A \cup (B \cup C)$

(iii) $(A \cap B) \cap C = A \cap (B \cap C)$ **(iv)** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(v) $(A \cup B)' = A' \cap B'$ **(Example)** **(vi)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(Example)**

29. For $A = \{1,2,3,4\}$, find the following relation in A. State the domain and range of each relation. Also draw the graph of each

(i) $\{(x, y)|y = x\}$ **(ii)** $\{(x, y)|\ y + x = 5\}$

(iii) $\{(x, y)|\ y + x < 5\}$ **(iv)** $\{(x, y)|\ y + x > 5\}$

30. Find the inverse of each of the following relation. Tell whether each relation and its inverse is a function or not

(i) $\{(2,1), (3,2), (4,3), (5,4), (6,5)\}$ **(ii)** $\{(x, y)|y = 2x + 3, x \in R\}$

(iii) $\{(x, y)|\ y^2 = 4ax, x \geq 0\}$ **(iv)** $\{(x, y)|\ x^2 + y^2 = 9, |x| \leq, |y| \leq 3\}$

31. Which of the following set have closure property w. r. t. addition and multiplication?

(i) $\{E, O\}$ **(ii)** $\{1, -1, i, -i\}$ **(iii)** $\{1, \omega, \omega^2\}$

32. Construct the truth table of **Residue classes Modulo 3,4,5 & 8** w. r. t. addition and multiplication

33. Write the properties of Binary Operations

34. What are the field axiom? In what respect does the field of real numbers differ from that of complex numbers?

35. State & prove **Cancellation Laws (Page No 76)**

36. If a, b being elements of a group G, solve the following equations

(i) $ax = b$ **(ii)** $xa = b$

37. If a, b are the elements of a group G, then show that $(ab)^{-1} = b^{-1}a^{-1}$

38. If $(G,*)$ is a group with e its identity, then e is unique.

39. Show that the set $\{1, \omega, \omega^2\}$, when $\omega^3 = 1$, is an Abelian group w. r. t. ordinary multiplication

40. If G is a group under the operation * and $a, b \in G$, find the solution of the equations

$$a * x = b, \quad x * a = b$$

41. Show that the set consisting of elements of the form $a + \sqrt{3}b$ (a, b being ational) is an Abelian group w. r. t. addition

42. Prove that all 2×2 non – singular matrices over the real field form a non – Abelian group under multiplication

Chapter No.03

Matrices & Determinants

• Short Questions

1. Definitions

(i)	Row Matrix	**(ii)**	Column Matrix	**(iii)**	Square Matrix
(iv)	Rectangular Matrix	**(v)**	Diagonal Matrix	**(vi)**	Scalar Matrix
(vii)	Equal Matrices	**(viii)**	Unit Matrix	**(ix)**	Multiplication of Matrices
(x)	Singular Matrix	**(xi)**	Non – Singular Matrix	**(xii)**	Upper Triangular Matrix
(xiii)	Triangular Matrix	**(xiv)**	Symmetric Matrix	**(xv)**	Lower Triangular Matrix
(xvi)	Skew Symmetric	**(xvii)**	Hermitian Matrix	**(xviii)**	Skew Harmitian Matrix
(xix)	Leading Entry	**(xx)**	Rank of Matrix		

2. Discus the cofactor of an element of a matrix or its determinants

3. Discus the minor of an element of a matrix or its determinants

4. Write any two properties of determinants

5. If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$

6. Find the value of 'x' and 'y' if

(i) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ **(ii)** $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

7. Find x and y if $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$

8. If $A = [a_{ij}]_{3\times 3}$, show that

(i) $\lambda(\mu A) = (\lambda\mu)A$ **(ii)** $(\lambda + \mu)A = \lambda A + \mu A$ **(iii)** $\lambda A - A = (\lambda - 1)A$

9. If $A = [a_{ij}]_{2\times 3}$ & $B = [b_{ij}]_{2\times 3}$, show that $\lambda(A+B) = \lambda A + \lambda B$

10. If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ & $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the value of a & b

11. If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ & $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of a & b

12. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then show that $(A+B)^t = A^t + B^t$

13. Find the matrix X if

(i) $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

14. If $A = [a_{ij}]_{3\times 4}$, show that

(i) $I_3A = A$ **(ii)** $AI_4 = A$

15. Find the inverse of the following matrices

(i) $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ **(iv)** $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

16. Solve the following system of linear equation

(i) $\left.\begin{matrix} 2x_1 - 3x_2 = 5 \\ 5x_1 + x_2 = 4 \end{matrix}\right\}$ **(ii)** $\left.\begin{matrix} 4x_1 + 3x_2 = 5 \\ 3x_1 - x_2 = 7 \end{matrix}\right\}$ **(iii)** $\left.\begin{matrix} 3x - 5y = 1 \\ -2x + y = -3 \end{matrix}\right\}$

17. If A and B are square matrices of the same order, then explain why in general

(i) $(A+B)^2 \neq A^2 + 2AB + B^2$ **(ii)** $(A-B)^2 \neq A^2 - 2AB + B^2$

(iii) $(A+B)(A-B) \neq A^2 - B^2$

18. Solve the following matrix equation for X

(i) $3X - 2A = B$ if $A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

(ii) $2X - 3A = B$ if $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

19. Find the cofactors A_{12}, A_{22} & A_{32} if $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$ and find $|A|$ **(Example)**

20. Evaluate the following determinants

(i) $\begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a+l & a-l & a \\ a & a+l & a-l \\ a-l & a & a+l \end{vmatrix}$ **(iii)** $\begin{vmatrix} 2a & a & a \\ b & 2b & b \\ c & c & 2c \end{vmatrix}$

21. Without expansion show that

(i) $\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$ **(ii)** $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$ **(iii)** $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$

22. Show that

(i) $\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = 9\begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix} = l^2(3a+l)$

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ **(iv)** $\begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix} = a^3 + b^3$

(v) $\begin{vmatrix} r\cos\varphi & 1 & -\sin\varphi \\ 0 & 1 & 0 \\ r\sin\varphi & 0 & \cos\varphi \end{vmatrix} = r$ **(vi)** $\begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix} = \lambda^2 (a+b+c+\lambda)$

23. If $A = \begin{vmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{vmatrix}, B = \begin{vmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & 2 \end{vmatrix}$, then find

(i) A_{12}, A_{22}, A_{32} & $|A|$ **(ii)** B_{21}, B_{22}, B_{23} & $|B|$

24. Without expansion verify that

(i) $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$ **(ii)** $\begin{vmatrix} 1 & 2 & 3x \\ 2 & 3 & 6x \\ 3 & 5 & 9x \end{vmatrix} = 0$

(iii) $\begin{vmatrix} 1 & a^2 & a/bc \\ 1 & b^2 & b/ca \\ 1 & c^2 & c/ab \end{vmatrix} = 0$ **(iv)** $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

(v) $\begin{vmatrix} bc & ca & ab \\ 1/a & 1/b & 1/c \\ a & b & c \end{vmatrix} = 0$ **(vi)** $\begin{vmatrix} mn & l & l^2 \\ nl & m & m^2 \\ lm & n & n^2 \end{vmatrix} = \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$

(vii) $\begin{vmatrix} 2a & 2b & 2c \\ a+b & 2b & b+c \\ a+c & b+c & 2c \end{vmatrix} = 0$ **(viii)** $\begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix} = 0$

25. Find the values of 'x' if

(i) $\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$ **(ii)** $\begin{vmatrix} 1 & x-1 & 3 \\ -1 & x+1 & 2 \\ 2 & -2 & x \end{vmatrix} = 0$ **(iii)** $\begin{vmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 3 & 6 & x \end{vmatrix} = 0$

26. Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$

27. If A is square matrix of order 3, then show that $|KA| = K^3|A|$

28. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ verify that $(A^{-1})^t = (A^t)^{-1}$

29. If A and B are non – singular matrices, then show that

(i) $(AB)^{-1} = B^{-1}A^{-1}$ **(ii)** $(A^{-1})^t = A$

30. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$, show that

(i) $A + A^t$ is symmetric **(ii)** $A - A^t$ is skew – symmetric

31. If A is any square matrix of order 3, show that

(i) $A + A^t$ is symmetric **(ii)** $A - A^t$ is skew – symmetric

32. If the matrices A and B are symmetric and AB = BA, show that AB is symmetric

33. Show that AA^t and A^tA are symmetric for any matrix of order 2×3

34. If $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$, show that

(i) $A + (\overline{A})^t$ is Hermitian **(ii)** $A - (\overline{A})^t$ is skew – Hermitian

35. If A is symmetric or skew – symmetric, show that A^2 is symmetric

36. If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, find $A(\overline{A})^t$

Long Questions

1. Find the matrix A if

(i) $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$

2. Show that $\begin{bmatrix} r\cos\varphi & 0 & -\sin\varphi \\ 0 & r & 0 \\ r\sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -r\sin\varphi & 0 & r\cos\varphi \end{bmatrix} = rI_3$

3. Without expansion, show that $\begin{vmatrix} x & a+x & b+c \\ x & b+x & c+a \\ x & c+x & a+b \end{vmatrix} = 0$ **(Example)**

4. Solve the equation $\begin{vmatrix} x & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 1 & 2 & 3 & 4 \\ -2 & x & 1 & -1 \end{vmatrix} = 0$ **(Example)**

5. Find A^{-1} if $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ (Page 109)**Example**

6. Show that

(i) $\begin{vmatrix} a & b+c & a+b \\ b & c+a & b+c \\ c & a+b & c+a \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(iii) $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

7. Evaluate the following determinants

(i) $\begin{vmatrix} 3 & 4 & 2 & 7 \\ 2 & 5 & 0 & 3 \\ 1 & 2 & -3 & 5 \\ 4 & 1 & -2 & 6 \end{vmatrix}$ (ii) $\begin{vmatrix} -3 & 9 & 1 & 1 \\ 0 & 3 & -1 & 2 \\ 9 & 7 & -1 & 1 \\ -2 & 0 & 1 & -1 \end{vmatrix}$

8. Find the values λ if A and B are singular $A = \begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & 1 & 2 & 0 \\ 8 & 2 & 5 & 1 \\ 3 & 2 & 0 & 1 \\ 2 & \lambda & -1 & 3 \end{bmatrix}$

9. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$ and show that $A^{-1}A = I_3$

10. Reduce the following matrix to (row) echelon and reduced (row) echelon form

$\begin{bmatrix} 2 & 3 & -1 & 9 \\ 1 & -1 & 2 & -3 \\ 3 & 1 & 3 & 2 \end{bmatrix}$ (**Example**) Page 122

11. Find the inverse of the following matrices. Also find there inverse by using row and column operation

(i) $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

12. Find the rank of the following matrices

(i) $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & -1 & 3 & 0 & -1 \\ 1 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & 2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{bmatrix}$

13. Use matrices to solve the system $\left.\begin{aligned} x_1 - 2x_2 + x_3 &= -4 \\ 2x_1 - 3x_2 + 2x_3 &= -6 \\ 2x_1 + 2x_2 + x_3 &= 5 \end{aligned}\right\}$ **(Example) (Page 133)**

14. Use Crammer's rule solve the system $\left.\begin{matrix} 3x_1 + x_2 - x_3 = -4 \\ x_1 + x_2 - 2x_3 = -4 \\ -x_1 + 2x_2 - x_3 = 1 \end{matrix}\right\}$ **(Example) (Page 137)**

15. Solve the following system of linear equations by Crammer's rule

(i) $\left.\begin{matrix} 2x + 2y + z = 3 \\ 3x - 2y - 2z = 1 \\ 5x + y - 3z = 2 \end{matrix}\right\}$ **(ii)** $\left.\begin{matrix} 2x_1 - x_2 + x_3 = 5 \\ 4x_1 + 2x_2 + 3x_3 = 8 \\ 3x_1 - 4x_2 - x_3 = 3 \end{matrix}\right\}$ **(iii)** $\left.\begin{matrix} 2x_1 - x_2 + x_3 = 8 \\ x_1 + 2x_2 + 2x_3 = 6 \\ x_1 - 2x_2 - x_3 = 1 \end{matrix}\right\}$

16. Use the matrices to solve the following system

(i) $\left.\begin{matrix} x - 2y + z = -1 \\ 3x + y - 2z = 4 \\ y - z = 1 \end{matrix}\right\}$ **(ii)** $\left.\begin{matrix} 2x_1 + x_2 + 3x_3 = 3 \\ x_1 + x_2 - 2x_3 = 0 \\ -3x_1 - x_2 + 2x_3 = -4 \end{matrix}\right\}$ **(iii)** $\left.\begin{matrix} x + y = 2 \\ 2x - z = 1 \\ 2y - 3z = -1 \end{matrix}\right\}$

17. Solve the following system by reducing their augmented matrices to the echelon form and their reduced echelon forms

(i) $\left.\begin{matrix} x_1 - 2x_2 - 2x_3 = -1 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 5x_1 - 4x_2 - 3x_3 = 1 \end{matrix}\right\}$ **(ii)** $\left.\begin{matrix} x + 2y + z = 2 \\ 2x + y + 2z = -1 \\ 2x + 3y - z = 9 \end{matrix}\right\}$ **(iii)** $\left.\begin{matrix} x_1 + 4x_2 + 2x_3 = 2 \\ 2x_1 + x_2 - 2x_3 = 9 \\ 3x_1 + 2x_2 - 2x_3 = 12 \end{matrix}\right\}$

18. Find the value of λ for which the following system does not possess a unique solution.

Also solve the system for the value of λ. $\left.\begin{matrix} x_1 + 4x_2 + \lambda x_3 = 2 \\ 2x_1 + x_2 - 2x_3 = 11 \\ 3x_1 + 2x_2 - 2x_3 = 16 \end{matrix}\right\}$

Chapter No.04

Quadratic Equation

• Short Questions

1. Definitions

(i) Linear Equation **(ii)** Quadratic Equation **(iii)** Solution of Quadratic Equation
(iv) Reciprocal Equation **(v)** Radical Equation **(vi)** Extrancous Roots
(vii) Remainder Theorem **(viii)** Factor Theorem **(ix)** Simultaneous Equation

2. Solve the following equations by factorization

(i) $x^2 - 7x + 10 = 0$ **(ii)** $x^2 + 7x + 12 = 0$ **(iii)** $9x^2 - 12x - 5 = 0$
(iv) $x^2 - x = 2$ **(v)** $\frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}$ **(vi)** $x(x+7) = (2x-1)(x+4)$
(vii) $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$

3. Solve the following equations by completing the square

(i) $ax^2 + bx + c = 0$ **(ii)** $x^2 + 4x - 1085 = 0$
(iii) $x^2 - 3x - 648 = 0$ **(iv)** $2x^2 + 12x - 110 = 0$

4. Find the roots of the following equation by using quadratic formula

(i) $5x^2 - 13x + 6 = 0$ **(ii)** $4x^2 + 7x - 1 = 0$
(iii) $15x^2 + 2ax - a^2 = 0$ **(iv)** $16x^2 + 8x + 1 = 0$

5. Solve the following equations

(i) $x^{1/2} - x^{1/4} - 6 = 0$ (**Example**) **(ii)** $2^{2x} - 3.2^{x+2} + 32 = 0$ (**Example**)
(iii) $2x^{-2} - 10 = 3x^{-1}$ **(iv)** $x^{2/5} + 8 = 6x^{1/5}$
(v) $4.2^{2x+1} - 9.2^{x} + 1 = 0$ **(vi)** $2^{x} + 2^{-x+6} - 20 = 0$
(vii) $4^{x} - 3.2^{x+3} + 128 = 0$ **(viii)** $3^{2x-1} - 12.3^{x} + 81 = 0$

6. Discuss the cube roots of unity

7. Prove that sum of all three cube roots of unity is zero **OR** Prove that $1 + \omega + \omega^2 = 0$

8. Prove that the product of all the three cube roots of unity is unity **OR** Prove that $\omega^3 = 1$

9. Find the fourth roots of unity

10. Evaluate the following

(i) $(1 + \omega - \omega^2)^8$ **(ii)** $\omega^{28} + \omega^{29} + 1$ **(iii)** $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$
(iv) $\left(-1 + \sqrt{-3}\right)^5 + \left(-1 - \sqrt{-3}\right)^5$

11. Show that

(i) $x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$ (**Example**) Page 153
(ii) $\left(-1 + \sqrt{-3}\right)^4 + \left(-1 + \sqrt{-3}\right)^4 = -16$ (**Example**) Page 153
(iii) $x^3 - y^3 = (x - y)(x - \omega y)(x - \omega^2 y)$
(iv) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots 2n$ factors $= 1$

12. If ω is the root of $x^2 + x + 1 = 0$, show that its other root of ω^2 and prove that $\omega^3 = 1$

13. Prove that complex cube roots of – 1 are $\frac{1+\sqrt{3}i}{2}$ & $\frac{1-\sqrt{3}i}{2}$ and hence prove that $\left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^2 = -2$

14. If ω is a cube roots of unity, form an equation whose roots are 2ω and $2\omega^2$

15. Find the four fourth roots 16, 81, 625

16. Solve the following equations
(i) $2x^4 - 32 = 0$ (ii) $3y^5 - 243y = 0$
(iii) $x^3 + x^2 + x + 1 = 0$ (iv) $5x^5 - 5x = 0$

17. Find numerical value of 'k' if the polynomial $x^3 + kx^2 - 7x + 6$ has a remainder of – 4, when divided by $x + 2$ (**Example**) Page 157

18. Show that $x - 2$ is a factor of $x^4 - 13x^2 + 36$ (**Example**) Page 158

19. If $x - 2$ and $x + 2$ are factors of $x^4 - 13x^2 + 36$. Using synthetic division, find the other two factors **Example**) Page 159

20. Use the reminder theorem to find the reminder when the first polynomial is divided by the second polynomial
(i) $x^2 + 3x + 7, x + 1$ (ii) $x^3 - x^2 + 5x + 4, x - 2$

21. Use the factor theorem to determine if the first polynomial is a factor of the second polynomial
(i) $x - 1, x^2 + 4x - 5$ (ii) $x - 2, x^3 + x^2 - 7x + 1$
(iii) $\omega + 2; 2\omega^3 + \omega^2 - 4\omega + 7$ (iv) $x - a; x^n - a^n$ where n is a positive
(v) $x + a; x^n + a^n$ where n is an odd

22. When $x^4 + 2x^3 + kx^2 + 3$ is divided by $x - 2$, the remainder is 1. Find the value of k

23. When the polynomial $x^3 + 2x^2 + kx + 4$ is divided by x – 2, the remainder is 14. Find the value of k.

24. Use the synthetic division to show that x is the solution of the polynomial and use the result to factorize the polynomial completely
(i) $x^3 - 7x + 6 = 0; x = 2$ (ii) $x^3 - 28x - 48 = 0, x = -4$
(iii) $4x^4 + 7x^3 - 4x^2 - 27x - 18, x = 2, x = 3$

25. Find the condition that one root of $ax^2 + bx + c = 0, a \neq 0$ is square of the other

26. If α, β are the roots of the $3x^2 - 2x + 4 = 0$, find the values of
(i) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (iii) $\alpha^3 + \beta^3$ (iv) $\alpha^2 - \beta^2$

27. If α, β are the roots of the $x^2 - px - p - c = 0$, prove that $(1 + \alpha)(1 + \beta) = 1 - c$

28. Find the condition that one root of $x^2 + px + q = 0$ is
(i) Double the other (ii) Square of the other
(iii) Additive inverse of the other (iv) Multiplicative inverse of the other

29. If the roots of the equation $x^2 - px + q = 0$ diff by unity, prove that $p^2 = 4q + 1$

30. Find the condition $\frac{a}{x-a} + \frac{b}{x-b} = 5$ may have roots equal in magnitude but opposite in signs

31. If α, β are the roots of the equation $ax^2 + bx + c = 0$, form the equations whose roots are
(i) α^2, β^2 (ii) $\frac{1}{\alpha}, \frac{1}{\beta}$ (iii) α^3, β^3

32. Discuss the nature of the roots of the following equation
(i) $x^2 + 2x + 3 = 0$ (**Example**) (ii) $2x^2 - 7x + 3 = 0$ (**Example**)
(ii) $x^2 - 5x + 6 = 0$ (iv) $2x^2 - 5x + 1 = 0$
(v) $25x^2 - 30x + 9 = 0$ (vi) $x^2 - 5x + 6 = 0$

33. Show that the roots of the equation $x^2 - 2\left(m - \frac{1}{m}\right)x + 3 = 0$ will be real

34. Show that the roots of the following equations will be rational

(i) $(p+q)x^2 - px - xq = 0$ **(ii)** $px^2 - (p-q)x - q = 0$

35. The sum of a positive number and its square is 380. Find the the number

36. The sum of a positive number and its reciprocal is $^{26}/_{5}$. Find the number

37. Find two consecutive numbers, whose product is 132

38. A number exceed its square root by 56. Find the number

Long Questions

1. Solve the following equations

(i) $8x^6 - 19x^3 - 27 = 0$ **(ii)** $(x-1)(x+5)(x+8)(x+2) - 880 = 0$

(iii) $(x+1)(2x+3)(2x+5)(x+3) = 945$ **(iv)** $(x^2+6x+8)(x^2+14x+48) = 105$

(v) $\left(x - \frac{1}{x}\right)^2 + 3\left(x + \frac{1}{x}\right) = 0$ **(vi)** $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

(vii) $x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0$

2. Solve the radical equations (**Examples**) Page 147 – 150

(i) $3x^2 + 15x - 2\sqrt{x^2+5x+1} = 2$ **(ii)** $\sqrt{3x^2-7x-30} - \sqrt{2x^2-7x-5} = x - 5$

3. Solve the following Equations (**Exercise 4.3**)

(i) $3x^2 + 2x - \sqrt{3x^2+2x-1} = 3$

(ii) $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2-3x+2}$

(iii) $\sqrt{2x+8} + \sqrt{x+5} = 7$

(iv) $\sqrt{2x^2-5x-3} + \sqrt{2x+1} = \sqrt{2x^2+25x+12}$

(v) $(x+4)(x+1) = \sqrt{x^2+2x-15} + 3x + 31$

(vi) $\sqrt{5x^2+7x+2} - \sqrt{4x^2+7x+18} = x - 4$

4. Show that $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$

5. If $x+1$ & $x-2$ are the factors of $x^3 + px^2 + qx + 2$ by using the synthetic division find the value of p and q

6. Use the synthetic division to find the values of p and q if $x+1$ & $x-2$ are the factors of the polynomial $x^3 + px^2 + qx + 6$

7. Find the value of 'a' and 'b' if – 2 and 2 are the roots of the polynomial $x^3 - 4x^2 + ax + b$

8. If the roots of $px^2 + qx + q = 0$ are α & β then prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

9. If α, β are the roots of the equation $ax^2 + bx + c = 0$, form the equations whose roots are

(i) $\frac{1}{\alpha^3}, \frac{1}{\beta^3}$ **(ii)** $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$ **(iii)** $-\frac{1}{\alpha^3}, -\frac{1}{\beta^3}$ **(iv)** $(\alpha-\beta)^2, (\alpha+\beta)^2$

10. If α, β are the roots of $5x^2 - x - 2 = 0$, from the equation whose roots are $\frac{3}{\alpha}$ & $\frac{3}{\beta}$

11. If α, β are the roots of $x^2 - 3x + 5 = 0$, from the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ & $\frac{1-\beta}{1+\beta}$

12. Show that the roots of the following equation are real
$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$ Also show that the roots will be equal only if $a = b = c$ (**Example**) Page 167

13. Show that the roots of $x^2 + (mx+c)^2 = a^2$ will be equal, if $c^2 = a^2(1+m^2)$

14. Show that the roots of $(mx+c)^2 = 4ax$ will be equal, if $c = \frac{a}{m}; m \neq 0$

15. Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots, if $c^2 = a^2m^2 + b^2; a \neq 0, b \neq 0$

16. Show that the roots of the equation $(a^2 - bc)x^2 + 2(b^2 - ca)x + c^2 - ab = 0$ will be equal, if either $a^3 + b^3 + c^3 = 3abc$ or $b = 0$

17. Solve the following systems of the equations

(i) $2x - y = 4; 2x^2 - 4xy - y^2 = 6$ (ii) $x + y = 5; x^2 + 2y^2 = 17$

(iii) $x + y = 5; \frac{2}{x} + \frac{3}{y} = 2, x \neq 0, y \neq 0$ (iv) $x + y = a + b; \frac{a}{x} + \frac{b}{y} = 2$

(v) $3x + 4y = 25; \frac{3}{x} + \frac{4}{y} = 2$

18. Solve the systems of Equations

(i) $2x^2 - 8 = 5y^2; x^2 - 13 = -2y^2$ (ii) $x^2 - 5xy + 6y^2 = 0; x^2 + y^2 = 45$

(iii) $12x^2 - 11xy + 2y^2 = 0; 4x^2 + 7y^2 = 148$

Chapter No.05

Partial Fractions

• Short Questions

1. Definitions

(i) Fraction (ii) Partial Fraction

(ii) Equation (iv) Identity

(v) Conditional Equation (vi) Rational Fraction

(vii) Proper Rational Fraction (viii) Improper Rational Fractional

2. Resolution of $\frac{P(x)}{Q(x)}$ into partial fractions when Q (x) has only non – repeated linear factors

3. Resolution of $\frac{P(x)}{Q(x)}$ into partial fractions when Q (x) has only repeated linear factors

4. Resolution of $\frac{P(x)}{Q(x)}$ into partial fractions when Q (x) has only non – repeated irreducible quadratic factors

5. Resolution of $\frac{P(x)}{Q(x)}$ into partial fractions when Q (x) has only repeated irreducible quadratic factors:

6. Resolve fraction $\frac{7x+25}{(x+3)(x+4)}$ into partial fraction

7. Resolve improper fraction $\frac{2x^3+x^2-x-3}{x(2x+3)(x-1)}$ into proper fraction.

8. Resolve fraction $\frac{1}{x^2-1}$ into partial fraction

9. Resolve fraction $\frac{x^2+1}{(x+1)(x-1)}$ into partial fraction

10. Resolve improper fraction $\frac{6x^3+x^2-5x+3}{2x^3+x^2-3x}$ into proper fraction.

11. Resolve fraction $\frac{x^2+1}{(x+1)(x-1)}$ into partial fraction without finding values of constant

12. Resolve $\frac{2x+1}{(x-1)(x+2)(x+3)}$ into partial fraction

13. Resolve fraction $\frac{7x+25}{(x+3)(x+4)}$ into partial fraction

14. If $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$, find value of B

15. For the identity $\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{3x-1}$ calculate the value of A

16. Write $\frac{1}{(1-ax)(1-bx)(1-cx)}$ into partial fraction without finding the value of constants

17. Resolve into partial fraction $\frac{2x+1}{(x+2)(x+3)(x-1)}$ statement incomplete

18. In the identity $7x + 25 = A(x + 4) + B(x + 3)$, calculate values of A and B

19. Resolve $\frac{9}{(x+2)^2(x-1)}$ into partial fraction

20. Write in mixed form (Proper Fraction) $\frac{3x^2+1}{x-1}$
21. Write identity for $\frac{x^2+x-1}{(x+2)^2}$
22. For the identity $\frac{1}{(x+1)^2(x^2-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$. Calculate the value of A & D
23. Resolve into partial fractions, without finding constants $\frac{x-1}{(x-2)(x+1)^3}$
24. Write $\frac{x^2}{(x-2)(x-1)^2}$ in form of partial fraction without finding constant
25. Write only into partial fractions form the expression $\frac{4x^3}{(x^2-1)(x+1)^2}$
26. Write only into partial fractions form $\frac{x^2}{(x-1)^3(x+1)}$. Not find the values of constants
27. Resolve $\frac{3x-11}{(x^2+1)(x+3)}$ into partial fractions
28. Write the partial fraction form of $\frac{8x^2}{(x^2+1)^2(1-x^2)}$ without finding values of constants
29. Write partial fraction form $\frac{2x^4-3x^3-4x}{(x^2+2)^2(x+1)^2}$
30. Write in mixed form $\frac{6x^3+5x^2-7}{2x^2-x-1}$
31. Resolve $\frac{4x}{(x^2+1)^2(x-1)}$ into partial fractions without finding the constant
32. Resolve $\frac{3x+7}{(x^2+4)(x+3)}$ into partial fractions without finding the constant
33. Write only partial fraction form of $\frac{x^2+1}{x^3+1}$ without finding constants
34. Resolve into partial fractions, without finding constants $\frac{x^2+15}{(x^2+2x+5)(x-1)}$
35. Write only partial fraction form of $\frac{x^2}{(x^2+1)^2(x-1)}$
36. Write partial fraction form of $\frac{4x^2+8x}{x^4+2x^2+9}$
37. Write identity for $\frac{x^2-2x+3}{x^4+x^2+1}$

Long Questions

1. Resolve the following into partial fraction (**Exercise 5.1**)
   (i) $\frac{6x^3+5x^2-7}{2x^2-x-1}$ (ii) $\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)}$
   (iii) $\frac{x}{(x-a)(x-b)(x-c)}$ (iv) $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$
2. Resolve the following into partial fraction (**Exercise 5.2**)
   (i) $\frac{5x^2-2x+3}{(x+2)^3}$ (ii) $\frac{4x}{(x+1)^2(x-1)}$
   (iii) $\frac{x^2}{(x-2)(x+1)^2}$ (iv) $\frac{1}{(x-1)^2(x+1)}$
   (v) $\frac{x-1}{(x-2)(x+1)^3}$ (vi) $\frac{2x^4}{(x-3)(x+2)^2}$
   (vii) $\frac{x^2+x-1}{(x+2)^3}$ (**Example**) Page 184
3. Resolve the following into partial fraction (**Exercise 5.3**)
   (i) $\frac{3x-11}{(x^2+1)(x+3)}$ (**Example**) Page 186 (ii) $\frac{9x-7}{(x^2+1)(x+3)}$

(iii) $\frac{x^2+1}{x^3+1}$ (iv) $\frac{x^4}{1-x^4}$

(v) $\frac{x^2-2x+3}{x^4+x^2+1}$

4. Resolve the following into partial fraction (**Exercise 5.4**)

(i) $\frac{4x^2}{(x^2+1)^2(x-1)}$ (**Example**)Page 188 (ii) $\frac{x^3+2x+2}{(x^2+x+1)^2}$

(iii) $\frac{x^2}{(x^2+1)^2(x-1)}$ (iv) $\frac{2x-5}{(x^2+2)^2(x-2)}$

Chapter No.06

Sequences and Series

• Short Questions

1. Definitions
   (i) Sequence (ii) Arithmetic Progression (iii) Geometric Progression
   (iii) Harmonic Progression
2. Find the sequence if $a_n - a_{n-1} + n + 1$ & $a_4 = 14$ (**Example**) Page 190
3. Write the first four terms of the sequence
   (i) $a_n = 2n - 3$ (ii) $a_n = (-1)^n n^2$
   (iii) $a_n = (-1)^n(2n-3)$ (iv) $a_n = \frac{n}{2n+1}$
   (v) $a_n = \frac{1}{2^n}$ (vi) $a_n - a_{n-1} = n + 2, a_1 = 2$
   (vii) $a_n = na_{n-1}, a_1 = 1$ (viii) $a_n = \frac{1}{a+(n-1)d}$
4. Find the indicated terms of the following sequences
   (i) $2,6,11,17, \ldots a_7$ (ii) $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \ldots a_7$
   (iii) $1,1,-3,5,-7,9 \ldots a_8$ (iv) $1,-3,5,-7,9,-11, \ldots a_8$
5. Find the next two term of the following sequences
   (i) 1,3,7,15,31, ... (ii) −1,2,12,40, ...
6. If $a_{n-2} = 3n - 11$, find the nth term of the sequence (**Example**) Page 194
7. Write the first four terms of the following arithmetic sequence, if $a_1 = 5$ and other three consecutive terms are 23,26,29
8. If $a_{n-3} = 2n - 5$, find the nth term of the sequence
9. Find the 13th term of the sequence $x, 1, 2-x, 3-2x, \ldots$
10. Which term of the A.P.
    (i) $5,2,-1, \ldots$ is -85? (ii) $-2,4,10, \ldots$ is 148?
11. If the nth term of the A.P. is 3n – 1, find the A.P
12. Determine whether −19 & 2 are the terms of the A.P. 17,13,9 .... Or not
13. Find the nth term of the sequence $\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots$
14. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. show that $b = \frac{2ac}{a+c}$
15. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. show that the common difference is $\frac{a-c}{2ac}$
16. Find A.M. between
    (i) $3\sqrt{5}$ & $5\sqrt{5}$ (ii) $x-3$ and $x+5$ (iii) $1-x+x^2$ & $1+x=x^2$
17. If 5,8 are two A.Ms between a and b, find ‘a’ and ‘b’
18. Find the 19th term and the partial sum of 19 terms of the arithmetic series
    $2 + \frac{7}{2} + 5 + \frac{13}{2} + \cdots$ (**Example**) Page 197
19. Find the sum of all the integral multiples of 3 between 4 and 97

20. Sum the series

(i) $-3+(-1)+1+3+5+\cdots+a_{16}$ **(ii)** $\frac{3}{\sqrt{2}}+2\sqrt{2}+\frac{5}{\sqrt{2}}+\cdots+a_{13}$

(iii) $-8-3\frac{1}{2}+1+\cdots+a_{11}$ **(iv)** $\frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}+\cdots$ to n term

21. How many terms of the series

(i) $-9-6-3+0+\cdots$ amount to 66? (**Example**) Page 198

(ii) $-7+(-5)+(-3)+\cdots$ amount to 65?

(iii) $-7+(-4)+(-1)+\cdots$ amount to 114?

22. Find the sum of 20th term of the series whose rth term is $3r+1$

23. If $S_n=n(2n-1)$, then find the series

24. Obtain the sum of all integers in the first 1000 integers which are neither divisible by 5 nor by 2

25. Find the 5th term of the G.P., $3, 6, 12, \ldots$

26. Find the 11th term of the sequence, $1+i, 2, \frac{4}{1+i}, \ldots$

27. Find the 12th term of $1+i, 2i, -2+2i, \ldots$

28. If an automobile depreciates in values 5% every year, at the end of 4 years what is the value of the automobile purchased for Rs. 12,000?

29. Which term of the sequence: $x^2-y^2, x+y, \frac{x+y}{x-y}, \ldots$ is $\frac{x+y}{(x-y)^2}$?

30. If a, b, c, d are in G.P., then prove that

(i) $a-b, b-c, c-d$ are in G.P **(ii)** $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

31. Show that the reciprocals of the terms of the geometric sequence $a_1, a_1r^2, a_1r^4, \ldots$ form another geometric sequence

32. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P. Show that the common ratio is $\pm\sqrt{\frac{a}{c}}$

33. Find the G.M. between

(i) -2 & 8 **(ii)** $-2i$ & $8i$

34. If both x and y are positive distinct real numbers, show that the geometric mean between x and y is less than their arithmetic mean

35. Find the sum of n terms of the geometric series if $a_n=(-3)\left(\frac{2}{5}\right)^n$ (**Example**) Page 210

36. Find the sum of the infinite G.P. $2, \sqrt{2}, 1 \ldots$ (**Example**) Page 213

37. If $\begin{matrix} a=1-x+x^2-x^3+\cdots & |x|<1 \\ b=1+x+x^2+x^3+\cdots & |x|<1 \end{matrix}$, then show that $2ab=a+b$ (**Example**) Page 214

38. Find the sum of the first 15 terms of the geometric sequence $1, {}^1/_3, {}^1/_9, \ldots$

39. Sum to n terms, the series

(i) $.2+.22+.222+\cdots$. **(ii)** $3+33+333+\cdots$

40. Sum the series $2+(1-i)+\left({}^1/_i\right)+\cdots$ to 8 terms

41. Find the sums of the following infinite geometric series

(i) ${}^1/_5+{}^1/_{25}+{}^1/_{125}+\cdots$ **(ii)** ${}^1/_2+{}^1/_4+{}^1/_8+\cdots$

(iii) ${}^9/_4+{}^3/_2+1+{}^2/_3+\cdots$ **(iv)** $4+2\sqrt{2}+2+\sqrt{2}+1+\cdots$

42. Find the vulgar fraction equivalent to the following recurring decimals

(i) $1.\dot{3}\dot{4}$ **(ii)** $0,\dot{7}$ **(iii)** $1.\dot{5}\dot{3}$

43. Find the sum to infinity of the series $r+(1+k)r^2+(1+k+k^2)r^3+\cdots$

44. If $y={}^x/_2+{}^{x^2}/_4+{}^{x^3}/_8+\cdots$ and if $0<x<2$, then prove that $x=\frac{2y}{1+y}$

45. If $y={}^x/_3+{}^{4x^2}/_9+{}^{8x^3}/_{27}+\cdots$ and if $0<x<\frac{3}{2}$, then prove that $x=\frac{3y}{2(1+y)}$

46. If $y=1+2x+4x^2+8x^3+\cdots$

(i) Show that $x = \frac{y-1}{2y}$ (ii) Find the interval in which the series is convergent

47. If $y = 1 + {}^{x}/_{2} + {}^{x^2}/_{4} + \cdots$
(i) Show that $x = 2\left(\frac{y-1}{y}\right)$ (ii) Find the interval in which the series is convergent

48. Find the H.M. between two numbers a and b

49. Find the 9th term of the harmonic sequence
(i) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \cdots$ (ii) $-\frac{1}{5}, -\frac{1}{3}, -1, \ldots$

50. Find the 12th terms of the following harmonic sequences
(i) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \cdots$ (ii) $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \cdots$

51. If 5 is the harmonic mean between 2 and b, find b

52. If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k

53. If A, G and H are arithmetic, geometric and harmonic mean between a and b respectively, show that $G^2 = AH$

54. Find A, G, H and verify that $A > G > H, (G > 0)$ IF
(i) $a = 2, b = 8$ (ii) $a = \frac{2}{5}, b = \frac{8}{5}$

55. Find A, G, H and verify that $A < G < H, (G < 0)$ IF
(i) $a = -2, b = -8$ (ii) $a = \frac{-2}{5}, b = \frac{-8}{5}$

56. Find A, G, H and show that $G^2 = AH$
(ii) $a = -2, b = -6$ (ii) $a = 2i, b = 4i$ (iii) $a = 9, b = 4$

57. **Theorem:** Prove that
(i) $\sum_{k=1}^{n} 1 = n$ (ii) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
(iii) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{2}$ (iv) $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$

Long Questions

1. If $l, m, n,$ are the pth, qth and rth term of an A.P. show that
(i) $l(q-r) + m(r-p) + n(p-q) = 0$ (ii) $p(m-n) + q(n-l) + r(l=m) = 0$
2. Find four A.Ms between $\sqrt{2}$ and ${}^{12}/_{\sqrt{2}}$
3. Find n so that ${}^{a^n + b^n}/_{a^{n-1} + b^{n-1}}$ may be the A.M. between a and b
4. Show that the sum of n A.Ms between a and b is equal to n times theirs A.M
5. Sum the series
(i) $3 + 5 - 7 + 9 + 11 - 13 + 15 + 17 - 19 + \cdots$ to 3n terms
(ii) $1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \cdots$ to 3n terms
6. If S_2, S_3, S_5 are the sums of 2n, 3n, 5n terms of an A.P. show that $S_5 = 5(S_3 - S_2)$
7. The sum of 9 terms of an A.P is 171 and its eight term is 31. Find the series
8. S_7 & S_9 are the sums of the first 7 and 9 terms of an A.P. respectively, if ${}^{S_9}/_{S_7} = {}^{18}/_{11}$ and $a_7 = 20$, find the series
9. The sum of the three numbers in an A.P. is 24 and their product is 440. Find the numbers
10. Find the four numbers in A.P. whose sum is 32 and the sum of whose squares is 276
11. If a^2, b^2 and c^2 are in A.P., show that ${}^{1}/_{b+c}, {}^{1}/_{c+a}, {}^{1}/_{a+b}$ are in A.P
12. Find the nth term of the geometric sequence if ${}^{a_5}/_{a_3} = {}^{4}/_{9}$ and $a_2 = {}^{4}/_{9}$
13. Find three, consecutive numbers in G.P. whose sum is 26 and their product is 216.

14. If three consecutive numbers in A.P. are increased by 1, 4, 15 respectively, the resulting numbers are in G.P. Find the original numbers if their sum is 6

15. Insert three G.Ms between
(i) 1 and 16 (ii) 2 and 32

16. For what value of n, $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the positive G.M between a & b?

17. The A.M. of two positive integral numbers exceeds their (Positive) G.M. by 2 and their sum is 20, find the numbers

18. The A.M. between two numbers is 5 and their (Positive) G.M. is 4. Find the numbers

19. What distance will a ball travel before coming to rest if it is dropped from a height of 75 meters and after each fall it rebounds $\frac{2}{5}$ of the distance it fell?

20. The sum of an infinite geometric series is 9 and the sum of the sequence of its terms is $\frac{81}{5}$. Find the series.

21. Insert four H.Ms between the following given numbers
(i) $\frac{1}{3}$ and $\frac{1}{23}$ (ii) $\frac{7}{3}$ and $\frac{1}{23}$ (iii) 4 and 20

22. The first term of an H.P. is $-\frac{1}{3}$ and the fifth term is $\frac{1}{5}$. Find its 9^{th} term

23. Find n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be H.M between a and b

24. If the H.M and A.M between two numbers are 4 and $\frac{9}{2}$ respectively, find the numbers

25. If the (Positive) G.M and H.M. between two numbers are 4 and $\frac{16}{5}$, find the numbers

26. Sum the following series up to n terms
(i) $1 \times 1 + 2 \times 4 + 3 \times 7 + \cdots$ (ii) $2 + (2+5) + (2+5+8) + \cdots$

27. Find the sum to n terms of the series whose nth terms are given
(i) $3n^2 + n + 1$ (ii) $n^2 + 4n + 1$

28. Given nth terms of the series, find the sum to 2n terms
(i) $3n^2 + 2n + 1$ (ii) $n^3 + 2n + 3$

Chapter No.07

Permutation, Combination and Probability

• Short Questions

1. Definitions
(i) Factorial (ii) Permutation
(iii) Circular Permutation (iv) Combination
(v) Probability (vi) Sample Space
(vii) Event (viii) Mutually Exclusive Event
(ix) Equally likely event

2. Evaluate each of the following
(i) $\frac{10!}{7!}$ (ii) $\frac{11!}{2!\,4!\,5!}$ (iii) $\frac{15!}{15!\,(15-15)!}$ (iv) $\frac{9!}{2!\,(9-2)!}$

3. Write each of the following in the factorial form
(i) $\frac{52.51.50.49}{4.3.2.1}$ (ii) $\frac{(n+1)(n)(n-1)}{3.2.1}$ (iii) $n(n-1)(n-2)$
(iv) $(n+2)(n+1)(n)$ (v) $n(n-1)(n-2)\ldots(n-r+1)$

4. How many signals can be made with 4 – different flags when any number of them are to be used at a time? (**Example**) Page 234

5. Evaluate the following
(i) ${}^{20}P_3$ (ii) ${}^{16}P_4$ (iii) ${}^{12}P_5$ (iv) ${}^{9}P_8$

6. Find the value of n when
(i) ${}^{n}P_3 = 30$ (ii) ${}^{11}P_n = 11.10.9$ (iii) ${}^{n}P_4 : {}^{n-1}P_3 = 9:1$

7. Prove from the first principle that

(i) ${}^{n}P_{r} = n.\,{}^{n-1}P_{r-1}$ (ii) ${}^{n}P_{r} = {}^{n-1}P_{r} + r.\,{}^{n-1}P_{r-1}$

8. How many signals can be given by 5 flags of different colours, using 3 flags at a time?
9. How many signals can be given by 6 flags of different colours when any number of flags can be used at a time?
10. How many words can be formed from the letters of the following words using all letters when no letter is to be repeated
    (i) PLANE (ii) OBJECT (iii) FASTING?
11. How many 3 – digit numbers can be formed by using each one the digits 2, 3, 5, 7, 9 only once?
12. How many 5 – digits multiples of 5 can be formed from the digits 2, 3, 5, 7, 9, when no digit is repeated
13. In how many ways can 5 boys and 4 girls be seated on a bench so that the girls and boys occupy alternate seats?
14. In how many ways can 5 persons be seated at a round table (**Example**) Page 237
15. In how many ways can a necklace of 8 beads of different coloures be made? (**Ex**) P 238
16. How many arrangements of the letters of the following words, taken all together, can be made
    (i) PAKPATTAN (ii) PAKISTAN
    (iii) MATHEMATICS (iv) ASSASSINATION
    (v) MISSISSIPPI (**Example**) P 237?
17. How many permutation of the letters of the word **PANAMA** can be made if P is to be the first letter in each arrangement?
18. How many arrangements of the letters of the word **ATTACKED** can be made if each arrangement begins with C and ends with K?
19. How many no's greater than 1000,000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4?
20. How many 6 – digit numbers can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them will lie between 4000,000 and 430,000?
21. In how many ways can 4 keys be arranged on a circular key ring?
22. How many necklaces can be made from 6 beads of different colour?
23. Prove that ${}^{n}C_{r} = {}^{n}C_{r-1}$ (**Complementary Combination**) Page 240
24. If ${}^{n}C_{8} = {}^{n}C_{12}$, find n (**Example**) Page 241
25. Find the number of the diagonals of a 6 – sided figure (**Example**) Page 241
26. Prove that ${}^{n-1}C_{r} + {}^{n-1}C_{r-1} = {}^{n}C_{r}$ (**Example**) Page 241
27. Evaluate the following
    (i) ${}^{12}C_{3}$ (ii) ${}^{20}C_{17}$ (iii) ${}^{n}C_{4}$
28. Find the value of n
    (i) ${}^{n}C_{5} = {}^{n}C_{4}$ (ii) ${}^{n}C_{10} = \frac{12\times 11}{2!}$ (iii) ${}^{n}C_{12} = {}^{n}C_{6}$
29. How many (a) Diagonal and (b) Triangles can be formed by joining the vertices of the polygon having
    (i) 5 sides (ii) 8 sides (iii) 12 sides?
30. The members of a club re 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?
31. In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?
32. Show that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$
33. There are 8 men and 10 women members of a club. How many committees of numbers can be formed, having
    (i) 4 Women (ii) At the most 4 women (iii) At least 4 women?

34. Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$
35. A die is rolled. What is the probability that the dots on the top are greater than 4? (**Example**) Page 244
36. What is the probability that a slip of numbers divisible by 4 are picked from the slips having numbers 1, 2, 3, ….,10? (**Example**) Page 245
37. **Experiment:** From a box containing orange – flavoured sweets, Bilal takes out one sweet without looking. Event Happening
    (i) The sweet is orange – flavoured (ii) The sweet is lemon – flavoured
38. **Experiment:** Pakistan and India play a circket match. The result is Event Happening
    (i) Pakistan Wins (ii) India does not lose
39. **Experiment:** There are 5 green and 3 red balls in a box, one ball is taken out Event Happing
    (i) The Ball is green (ii) The ball is red
40. **Experiment:** A fair coin is tossed three times. It shows Event Happening
    (i) One tail (ii) At least one head
41. **Experiment:** A die is rolled. The top shows Event Happening
    (i) 3 or 4 dots (ii) Dots less than 5
42. **Experiment:** Two dice, one red and the blue, are rolled simultaneously. The numbers of dots on the topes are added. The total of the two scores is Event Happening
    (i) 5 (ii) 7 (iii) 11

Long Questions

1. Find the numbers greater than 23000 that can be formed form the digits 1, 2, 3, 5, 6, without repeating any digit
2. Find the number of 5 – digit numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but
    (i) The digits 2 and 8 are next to each other
    (ii) The digits 2 and 8 are not nxt to each other
3. The D.C.Os of 11 districts meet to discuss the law and order situation in their districts. In how many ways can they be seated at around table, when two particular D.C.Os insist on sitting together?
4. Find the values of n and r, when
    (i) ${}^{n}C_{r} = 35$ and ${}^{n}P_{r} = 210$ (ii) ${}^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3 : 6 : 11$
5. A die is thrown. The probability that the dots on the top are prime numbers or odd numbers (**Example**) Page 250
6. If sample space $= \{1,2,3,\ldots,9\}$. Event $A = \{2,4,6,8\}$ & $B = \{1,3,5\}$, find $P(A \cup B)$.
7. A die is thrown twice. What is the probability that the sum of the number of dots shown is 3 or 11?
8. Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?
9. Two dice are thrown. E_1 is the event that sum of their dots is an odd number and E_2 is the event that 1 is the dot on the top of the first die. Show that $P(E_1 \cap E_2) = P(E_1).P(E_2)$.
10. A die is rolled twice, Event E_1 is the appearance of even number of the dots and event E_2 is the appearance of more than 4 dots. Prove that $P(E_1 \cap E_2) = P(E_1).P(E_2)$.
11. Find the probability that sum of dots appearing in two successive throws of two dice is every time 7
12. A fair die is thrown twice. Find the probability that a prime number pf dots appear in the first throw and the number of dotes in the second throw is less than 5

Chapter No.08

Mathematical Inductions and Binomial Theorem

• Short Questions

1. Definitions
   (i) Principle of Mathematical Induction (ii) Principle of Extended Mathematical Induction
2. Show that $\frac{n^3+2n}{3}$ represents an integer $\forall\ n \in N$ (**Example**) Page 259
3. Show that the inequality $4^n > 3^n + 4$ is true, for integral values of $n \geq 2$ (**Ex**) Page 262
4. Prove by mathematical induction that for all positive integral values of n
   (i) $n^2 + n$ is divisible by 2 (ii) $5^n - 2^n$ is divisible by 3
   (iii) $5^n - 1$ is divisible by 4 (iv) $8 \times 10^n - 2$ is divisible by 6
   (v) $n^3 - n$ is divisible by 6
5. Use mathematical induction to prove the following formulae for every positive integer n
   (i) $x + 1$ is a factor of $x^{2n} - 1$; $(x \neq -1)$
   (ii) $x - y$ is a factor of $x^n - y^n$; $(x \neq y)$
   (iii) $x + y$ is a factor of $x^{2n-1} + y^{2n-1} (x \neq -y)$
6. Use the principle of extended mathematical induction to prove that
   (i) $n! > 2^n - 1$ for integral values of $n \geq 4$
   (ii) $n^2 > n + 3$ for integral values of $n \geq 3$
   (iii) $4^n > 3^n + 2^{n-1}$ for integral values of $n \geq 2$
   (iv) $3^n < n!$ for integral values of $n > 6$
   (v) $n! > n^2$ for integral values of $n \geq 4$
   (vi) $1 + nx \leq (1 + x)^n$ for $n \geq 2$ and $x > -1$
7. State the Binomial Theorem & Binomial Series
8. Evaluate $(9.9)^5$ (**Example**) Page 269
9. Find the specified term in the expansion of $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{11}$ (**Example**) Page 269
   (i) The term involving x^5 (ii) The fifth term
   (iii) The sixth term from the end (iv) Coefficient of term involving x^{-1}
10. Find the following in the expansion of $\left(\frac{x}{2} + \frac{2}{x^2}\right)^{12}$ (**Example**) Page 270
    (i) The term independent of x (ii) The middle term
11. Show that $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n.2^{n-1}$ (**Example**) Page 272
12. Using the Binomial theorem, expand the following
    (i) $(a + 2b)^5$ (ii) $\left(\frac{x}{2} - \frac{2}{x^2}\right)^6$ (iii) $\left(3a - \frac{x}{3a}\right)^4$
    (iv) $\left(\frac{x}{2y} - \frac{2y}{x}\right)^8$ (v) $\left(\sqrt{\frac{a}{x}} - \sqrt{\frac{x}{a}}\right)^6$
13. Calculate the following by means of binomial theorem
    (i) $(0.97)^3$ (ii) $(2.03)^4$ (iii) $(9.98)^4$ (iv) $(21)^5$
14. Find the term involving
    (i) x^4 in the expansion of $(3 - 2x)^7$ (ii) x^{-2} in the expansion of $\left(x - \frac{2}{x^2}\right)^{13}$
    (iii) a^4 in the expansion of $\left(\frac{2}{x} - a\right)^9$ (iv) y^4 in the expansion of $\left(x - \sqrt{y}\right)^{11}$
15. Find the coefficient of
    (i) x^5 in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$ (ii) n in the expansion of $\left(x^2 - \frac{1}{x}\right)^{2n}$
16. Find the 6th term in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$
17. Find the term independent of x in the following expansions

(i) $\left(x-\frac{2}{x}\right)^{10}$ (ii) $\left(\sqrt{x}+\frac{1}{2x^2}\right)^{10}$ (iii) $(1+x^2)\left(1+\frac{1}{x^2}\right)^4$

18. Determine the middle term in the following expansions

(i) $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$ (ii) $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$ (iii) $\left(2x-\frac{1}{2x}\right)^{2m+1}$

19. Find $(2n+1)^{th}$ term from the end in the expansion of $\left(x-\frac{1}{2x}\right)^{3n}$

20. Evaluate $\sqrt[3]{30}$ correct to the three places of decimal

21. Expand the following up – to 4 terms, taking the values of x such that the expansion in each case is valid

(i) $(1-x)^{1/2}$ (ii) $(1+2x)^{-1}$ (ii) $(1+x)^{-1/3}$ (iv) $(4-3x)^{-1/3}$

(v) $(8-2x)^{-1}$ (vi) $(2-3x)^{-2}$ (vii) $\frac{\sqrt{1+2x}}{1-x}$

22. Using the binomial theorem find the value of the following to three places of decimals

(i) $\sqrt{99}$ (ii) $(1.03)^{1/3}$ (iii) $\sqrt[3]{65}$ (iv) $\sqrt[4]{17}$

(v) $\sqrt[5]{31}$ (vi) $1/\sqrt[5]{252}$ (vii) $\sqrt{7}/\sqrt{8}$

23. If x is so small that its square and higher powers can be neglected, then show that

(i) $\frac{1-x}{\sqrt{1+x}}\approx 1-\frac{3}{2}x$ (ii) $\frac{\sqrt{1+2x}}{\sqrt{1-x}}\approx 1+\frac{3}{2}x$

(iii) $\frac{\sqrt{4+x}}{(1-x)^3}\approx 2+\frac{25}{4}x$ (iv) $\frac{(1-x)^{1/2}(9-4x)^{1/2}}{(8+3x)^{1/3}}\approx \frac{3}{2}-\frac{61}{48}x$

24. If x is so small that its cube and higher powers can be neglected, then show that

(i) $\sqrt{1-x-2x^2}\approx 1-\frac{x}{2}-\frac{9x^2}{8}$ (ii) $\sqrt{\frac{1+x}{1-x}}\approx 1+x+\frac{1}{2}x^2$

Long Questions

1. Use mathematical induction to prove the following formulae for every positive integer n

(i) $1+5+9+\cdots+(4n-3)=n(2n-1)$

(ii) $1+3+5+\cdots+(2n-1)=n^2$

(iii) $1+2+4+\cdots+2^{n-1}=2^n-1$

(iv) $1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{n-1}}=2\left[1-\frac{1}{2^n}\right]$

(v) $2+6+18+\cdots+2\times 3^{n-1}=3^n-1$

(vi) $\frac{1}{2\times 5}+\frac{1}{5\times 8}+\frac{1}{8\times 11}+\cdots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{2(3n+2)}$

(vii) $r+r^2+r^3+\cdots+r^n=\frac{r(1-r^n)}{1-r}\ ;(r\neq 1)$

(viii) $1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(4n^2-1)}{3}$

(ix) $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\cdots+\binom{n+2}{3}=\binom{n+3}{4}$

(x) $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots+\frac{1}{3^n}=\frac{1}{2}\left[1-\frac{1}{3^n}\right]$

2. Expand $\left(\frac{a}{2}-\frac{2}{a}\right)^6$ and also find its general terms

3. Expand and simplify the following

(i) $\left(a+\sqrt{2}x\right)^4+\left(a-\sqrt{2}x\right)^4$ (ii) $\left(2+\sqrt{3}\right)^5+\left(2-\sqrt{3}\right)^5$

(iii) $(2+i)^5-(2-i)^5$ (iv) $\left(x+\sqrt{x^2-1}\right)^3+\left(x-\sqrt{x^2-1}\right)^3$

4. Show that the middle term of $(1+x)^{2n}$ is $\frac{1.3.5\ldots(2n-1)}{n!}2^n x^n$
5. Show that $\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\cdots+\binom{n}{n-1}=2^{n-1}$
6. Show that $\binom{n}{0}+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\frac{1}{4}\binom{n}{3}+\cdots+\frac{1}{n+1}\binom{n}{n}=\frac{2^{n+1}-1}{n+1}$
7. If m and n are nearly equal, show that $\left(\frac{5m-2n}{3n}\right)^{1/3}\approx\frac{m}{m+2n}+\frac{n+m}{3n}$ (**Example**) Page 279
8. For $y=\frac{1}{2}\left(\frac{4}{9}\right)+\frac{1}{2^2}.\frac{3}{2!}\left(\frac{4}{9}\right)^2+\frac{1.3.5}{2^3 3!}\left(\frac{4}{9}\right)^3+\cdots$ Show that $5y^2+10y-4=0$(**Ex**) Page 281
9. If x is very nearly equal 1, then prove that $px^p-qx^q\approx(p-q)x^{p+q}$
10. If p – q is small when compared with p or q, show that $\frac{(2n+1)p+(2n-1)q}{(2n-1)p+(2n+1)q}\approx\left(\frac{p+q}{2q}\right)^{1/n}$
11. Show that $\left[\frac{n}{2(n+N)}\right]^{\frac{1}{2}}\approx\frac{8n}{9n-N}-\frac{n+N}{4n}$ where n and N are nearly equal.
12. Use binomial theorem to show that $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+\cdots=\sqrt{2}$
13. If $y=\frac{1}{3}+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+\cdots$, then prove that $y^2+2y-2=0$
14. If $2y=\frac{1}{2^2}+\frac{1.3}{2!}.\frac{1}{2^4}+\frac{1.3.5}{3!}.\frac{1}{2^6}+\cdots$, prove that $4y^2+4y-1=0$
15. If $y=\frac{2}{5}+\frac{1.3}{2!}.\left(\frac{2}{5}\right)^2+\frac{1.3.5}{3!}.\left(\frac{2}{5}\right)^3+\cdots$, prove that $y^2+2y-4=0$

Chapter No.09

Fundamental of Trigonometry

• Short Questions

1. Definitions
   - (i) Angle
   - (ii) Degree
   - (iii) Radian
   - (iv) Co – terminal Angle
2. Covert $18^0\ 6'\ 21''$ to decimal form (**Example**) Page 287
3. Convert 21.256^0 to the $D^0M'S''$form (**Example**) Page 287
4. Prove that $\theta=\frac{l}{r}$
5. An arc subtends an angle of 70^0 at the center of a circle and its length is 132mm. Find the radius of the circle (**Example**) Page 290
6. Find the length of the equatorial arc subtending an angle of 1^0at the center of the earth, taking the radius of the earth as 64000km
7. Express the following sexagesimal measures of angles in radians
   (i) 150^0 (ii) 135^0 (iii) $35^0 20'$ (iv) $75^0 6' 30''$
   (v) $120^0 40'$ (vi) $154^0 20'$
8. Convert the following radian measures of angles into the measures of sexagesimal system
   (i) $11\pi/27$ (ii) $13\pi/16$ (iii) $17\pi/24$ (iv) $19\pi/32$
9. What is the circular measure of the angle between the hands of a watch at 4 O' clock?
10. Find "θ" if
    (i) $l=1.5$ cm ; $r=2.5$ cm (ii) $l=3.2$ m ; $r=2$ m
11. Find "l" if
    (i) $\theta=\pi$ radian ; $r=6$ cm (ii) $\theta=65^0 20'$; $r=18$ mm
12. Find "r" if
    (i) $l=5$ cm ; $\theta=1/2$ radian (ii) $l=56$ cm ; $\theta=45^0$

13. What is the length of the arc intercepted on a circle of radius 14 cms by the arms of a central angle of 45^0?

14. Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35 cm.

15. A railway train is running on a circular track of radius 500 meters at the rate of 30 km per hour. Through what angle will it turn in 10 sec

16. Show that the area of sector of a circular region of radius r is $^1/_2\, r^2\theta$, where θ is the circular measure of the central angle of the sector.

17. Two cities A and B lie on the equator such that their longitudes are 45^0E and 25^0W respectively. Find the distance between the two cities, taking radius of the earth as 6400kms

18. Prove that (**Fundamental Identities**)

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $1 + \cot^2\theta = \sec^2\theta$

19. In which quadrant are the terminal arms of the angle lie when

(i) $\sec\theta < 0$ and $\sin\theta < 0$ (ii) $\cos\theta < 0$ and $\tan\theta < 0$?

20. Find the values of the remaining trigonometric functions

(i) $\sin\theta = \frac{12}{13}$ and terminal arm of the angle is in Q – I

(ii) $\cos\theta = \frac{9}{41}$ and terminal arm of the angle is in Q – IV

(iii) $\tan\theta = -\frac{1}{3}$ and terminal arm of the angle is in Q – II

(iv) $\sin\theta = -\frac{1}{\sqrt{2}}$ and terminal arm of the angle is not in Q – III

21. If $\cot\theta = \frac{15}{8}$ and terminal arm of the angle is not in Q – I, find the values of $\cos\theta$ & $\operatorname{cosec}\theta$

22. Verify the following

(i) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0 = \sin 30^0$

(ii) $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \sin^2\frac{\pi}{4} = 2$

(iii) $2\sin 45^0 + \frac{1}{2}\operatorname{cosec} 45^0 = \frac{3}{\sqrt{2}}$

(iv) $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4$

23. Evaluate the following

(i) $\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}.\tan\frac{\pi}{6}}$ (ii) $\dfrac{1 - \tan^2\frac{\pi}{3}}{1 + \tan^2\frac{\pi}{3}}$

24. Verify the following when $\theta = 30^0, 45^0$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$ (ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(iii) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

25. Find x, if $\tan^2 45^0 - \cos^2 60^0 = x.\sin 45^0 \cos 45^0 \tan 60^0$

26. Prove that $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta, \forall\, \theta \in R$ (**Example**) Page 310

27. Prove that $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$ where $A \neq \frac{n\pi}{2}; n \in Z$ (**Ex**) Page 310

28. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$, where θ is not an odd multiple of $\frac{\pi}{2}$ (**Ex**) Page 310

29. Show that $\cot^4\theta + \cot^2\theta = \operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta$, where θ is not an odd multiple of $\frac{\pi}{2}$

30. Prove that the following identities

(i) $\sec\theta.\operatorname{cosec}\theta \sin\theta.\cos\theta = 1$ (ii) $\cos\theta + \tan\theta.\sin\theta = \sec\theta$

(iii) $\operatorname{cosec}\theta + \tan\theta\sec\theta = \cos\theta\sec^2\theta$ (iv) $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

(v) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$ (vi) $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

(vii) $\cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(viii) $\frac{\sin\theta}{1+\cos\theta} + \cot\theta = \text{cosec}\,\theta$

(ix) $\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

(x) $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

(xi) $\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$

(xii) $(\tan\theta + \cot\theta)^2 = \sec^2\theta\,\text{cosec}^2\theta$

(xiii) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

(xiv) $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{2}{1-2\sin^2\theta}$

Long Questions

1. If $\text{cosec}\,\theta = \frac{m^2+1}{2m}$ and $m > 0\left(0 < \theta < \frac{\pi}{2}\right)$, find the remaining trigonometric ratios
2. If $\tan\theta = \frac{1}{\sqrt{7}}$ and terminal arm of the angle is not in Q – III, find the values of $\frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \text{sce}^2\theta}$
3. If $\cot\theta = \frac{5}{2}$ and terminal arm of the angle is in Q – I, find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$
4. Find the values of the trigonometric function of the following Quadrantal angles
   (i) $\frac{5\pi}{2}$ (ii) $-\frac{9}{2}\pi$ (iii) 1530^0 (iv) -2430^0
   (v) $\frac{235}{2}\pi$ (vi) $\frac{407}{2}\pi$ (vii) 390^0 (viii) -330^0
   (ix) $-\frac{17}{3}\pi$ (x) $-\frac{71}{6}\pi$ (xi) -1035^0
5. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \tan\theta + \sec\theta$
6. Prove that $\frac{1}{\text{cosec}\,\theta - \cot\theta} + \frac{1}{\sin\theta} = \frac{1}{\sin\theta} + \frac{1}{\text{cosec}\,\theta + \cot\theta}$
7. Prove that $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$
8. Prove that $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$
9. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Chapter No.10

Trigonometric Identities Sum and Difference of Angle

Short Questions

1. Define allied angle
2. Without using the tables, write down the values of (**Example**) Page 320
   (i) $\cos 315^0$ (ii) $\sin 540^0$ (iii) $\tan(-135^0)$ (iv) $\sec(-300^0)$
3. Simplify $\frac{\sin(360^0-\theta)\cos(180^0-\theta)\tan(180^0+\theta)}{\sin(90^0+\theta)\cos(90^0-\theta)\tan(360^0+\theta)}$
4. Without using the tables, find the values of
   (i) $\cot(-855^0)$ (ii) $\tan(1110^0)$ (iii) $\sin(-300^0)$
5. Prove that the following
   (i) $\sin(180^0+\alpha)\sin(90^0-\alpha) = -\sin\alpha\cos\alpha$
   (ii) $\sin 780^0 \sin 480^0 + \cos 120^0 \sin 30^0 = \frac{1}{2}$
   (iii) $\cos 306^0 + \cos 234^0 + \cos 162^0 + \cos 18^0 = 0$
   (iv) $\cos 330^0 \sin 600^0 + \cos 120^0 \sin 150^0 = -1$
6. If α, β, γ are the angles of a triangle ABC, then prove that
   (i) $\sin(\alpha+\beta) = \sin\gamma$
   (ii) $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$
   (iii) $\sin(\alpha+\beta) = -\cos\gamma$
   (iv) $\tan(\alpha+\beta) + \tan\gamma = 0$
7. Without using tables, find the values of all trigonometric functions of 75^0 (**Ex**) Page 322

8. Prove that $\tan 56^0 = \frac{\cos 11^0 + \sin 11^0}{\cos 11^0 - \sin 11^0}$

9. Find the values of the following

(i) $\tan 15^0$ (ii) $\sin 105^0$ (iii) $\cos 105^0$ (iv) $\tan 105^0$

10. Prove that

(i) $\sin(45^0 + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$ (ii) $\cos(45^0 + \alpha) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

11. Prove that

(i) $\tan(45^0 + A)\tan(45^0 - A) = 1$ (ii) $\tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right) = 0$

(iii) $\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right) = \cos\theta$ (iv) $\frac{\sin\theta - \cos\theta\tan\frac{\theta}{2}}{\cos\theta + \sin\theta\tan\frac{\theta}{2}} = \tan\frac{\theta}{2}$

(v) $\frac{1 - \tan\theta\tan\varphi}{1 + \tan\theta\tan\varphi} = \frac{\cos(\theta + \varphi)}{\cos(\theta - \varphi)}$

12. Show that $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \tan\alpha$

13. Prove that $\frac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0} = \tan 37^0$

14. Express the following in the form of measures of $r\sin(\theta + \varphi)$ or $r\sin(\theta - \varphi)$, where

(i) $3\sin\theta + 5\sin\theta = 13$ (ii) $\sin\theta - \cos\theta$

15. Prove that (**Double Angle Identities**)

(i) $\sin 2\alpha = 2\sin\alpha\cos\alpha$ (ii) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ (iii) $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

16. Prove that (**Triple Angle Identities**)

(i) $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ (ii) $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

(iii) $\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$

17. Prove that $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$ (**Example**) Page 330

18. Show that (**Example**) Page 331

(i) $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ (ii) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

19. Find the values of $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$ when

(i) $\sin\alpha = \frac{12}{13}$, where $0 < \alpha < \frac{\pi}{2}$ (ii) $\cos\alpha = \frac{3}{5}$, where $0 < \alpha < \frac{\pi}{2}$

20. Prove the following identities

(i) $\cot\alpha - \tan\alpha = 2\cot 2\alpha$ (ii) $\frac{\sin 2\alpha}{1 + \cos 2\alpha} = \tan\alpha$

(iii) $\sqrt{\frac{1 + \sin\alpha}{1 - \sin\alpha}} = \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}$ (iv) $\frac{\operatorname{cosec}\theta + 2\operatorname{cosec}2\theta}{\sec\theta} = \cot\frac{\theta}{2}$

(v) $\frac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta\,.\tan\theta$ (vi) $\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$

21. Prove without using tables/ calculator, that $\sin 19^0\cos 11^0 + \sin 71^0\sin 11^0 = \frac{1}{2}$ P 334

22. Express $\cos A + \cos 3A + \cos 5A + \cos 7A$ as a product

23. Express the following product as sums or differences

(i) $2\sin 3\theta\cos\theta$ (ii) $2\cos 5\theta\sin 3\theta$

(iii) $\sin 5\theta\cos\theta$ (iv) $\cos(x + y)\sin(x - y)$

(v) $\cos(2x + 30^0)\cos(2x - 30^0)$ (vi) $\sin(x + 45^0)\sin(x - 45^0)$

24. Express the following sums or differences as product

(i) $\sin 8\theta - \sin 4\theta$ (ii) $\cos 7\theta - \cos\theta$

(iii) $\cos 12^0 + \cos 48^0$ (iv) $\sin(x + 30^0) + \sin(x - 30^0)$

25. Prove that

(i) $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$ (ii) $\frac{\sin\alpha - \sin\beta}{\sin\alpha + \sin\beta} = \tan\left(\frac{\alpha-\beta}{2}\right)\cot\left(\frac{\alpha+\beta}{2}\right)$

26. Prove that

(i) $\cos 20^0 + \cos 100^0 + \cos 140^0 = 0$

(ii) $\sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\cos 2\theta$

(iii) $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

Long Questions

1. Prove that

(i) $\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\text{cosec}\,(2\pi-\theta)} = \cos\theta$

(ii) $\frac{\cos(90^0+\theta)\sec(-\theta)\tan(180^0-\theta)}{\sec(360^0-\theta)\sin(180^0+\theta)\cot(90^0-\theta)} = -1$

2. If α, β, γ are the angle of triangle ABC, then prove that (**Example**) Page 324

(i) $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$

(ii) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

3. Show that $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$

4. If $\sin\alpha = \frac{4}{5}$ & $\cos\beta = \frac{40}{41}$, where $0 < \alpha < \frac{\pi}{2}$ and $0 < \beta < \frac{\pi}{2}$ show that $\sin(\alpha+\beta) = \frac{133}{205}$

5. Find $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, given that

(i) $\tan\alpha = \frac{3}{4}, \cos\beta = \frac{5}{13}$ and neither the terminal side of the angle of measure α nor that β is in Q – I

(ii) $\tan\alpha = \frac{-15}{8}, \sin\beta = \frac{-7}{25}$ and neither the terminal side of the angle of measure α nor that β is in Q – IV

6. If α, β, γ are the angle of triangle ABC, then prove that $\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}.\cot\frac{\beta}{2}.\cot\frac{\gamma}{2}$

7. If $\alpha + \beta + \gamma = 180^0$, show that $\cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha = 1$

8. Reduce $\cos^4\theta$ to an expression involving only function of multiples of θ, raised to the first power (**Example**) Page 331

9. Reduce $\sin^4\theta$ to an expression involving only function of multiples of θ, raised to the first power

10. Show that $\cos 20^0\cos 40^0\cos 80^0 = \frac{1}{8}$

11. Prove that

(i) $\cos 20^0\cos 40^0\cos 60^0\cos 80^0 = \frac{1}{16}$

(ii) $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4}{9} = \frac{3}{16}$

(iii) $\sin 10^0\sin 30^0\sin 50^0\sin 70^0 = \frac{1}{16}$

Chapter No.11

Trigonometric Functions and their Graphs

- ## Short Questions

1. Definitions

(i) Periodicity (ii) Period of trigonometric functions

2. Prove that 2π is the period of sine
3. Prove that π is the period of tangent
4. Find the period of (**Example**) Page 341
   (i) $\sin 2x$ (ii) $\tan\frac{x}{3}$
5. Find the period of the following
   (i) $\cos 2x$ (ii) $\sin\frac{x}{5}$ (iii) $\tan\frac{x}{7}$ (iv) $\cot 8x$ **(v)** $3\cos\frac{x}{5}$

Chapter No.12

Application of Trigonometry

• Short Questions

1. Definitions
   (i) Angle of Elevation (ii) Angle of Depression
   (iii) Circum – Circle (iv) In – Circle
   (v) Escribed Circle
2. Solve the right triangle ABC, in which $\gamma = 90^0$
   (i) $\alpha = 58^0 13', b = 125.7$ **Ex (P 354)** (ii) $\alpha = 62^0 40', b = 796$
   (iii) $\alpha = 37^0 20', a = 243$ (iv) $a = 3.28, b = 5.74$
3. A vertical pole is 8 meter high and the length of its shadow is 6 meter. What is the angle of elevation of the sum at that moment?
4. A ladder leaning against a vertical wall makes an angle of 24^0 with the wall. Its foot is 5 meter from the wall. Find its length
5. A kite flying at height of 67.2 meter is attached to a fully stretched string inclined at an angle of $55^0$to the horizontal. Find the length of the string
6. When the angle between the ground and the sun is 30^0,flag pole casts a shadow of 40 meter long. Find the height of the top of the flag.
7. Write the following of half the angle in terms of sides
   (i) Sine (ii) Cosine (iii) Tangent
8. Solve the triangle ABC, given that
   (i) $\alpha = 35^0 17', \beta = 45^0 13', b = 421$ (ii) $\beta = 60^0, \gamma = 15^0, b = \sqrt{6}$
   (iii) $\beta = 52^0, \gamma = 89^0 35', a = 89.35$ (iv) $\alpha = 42^0 45', \gamma = 74^0 32', c = 16.1$
9. Solve the triangle ABC, given that
   (i) $a = \sqrt{3} - 1, b = \sqrt{3} + 1$ & $\gamma = 60^0$ (ii) $a = 3, bc = 6$ & $\beta = 36^0 20'$
10. Two forces of 40N and 30N are represented by $\overrightarrow{AB}$ and $\overrightarrow{BC}$ which are inclined at an angle of $147^0 25'$. Find $\overrightarrow{AC}$, the resultant of $\overrightarrow{AB}$ and $\overrightarrow{BC}$
11. Solve the following triangles, in which by using law of cosine or half angle formula
    (i) $a = 7, b = 7, c = 9$ (ii) $a = 4584, b = 5140, c = 3624$
12. Find the smallest angle of the triangle ABC, when $a = 37.34, b = 3.24, c = 35.06$
13. Find the measurement of the greatest angle, if sides of the triangle are 16, 20, 33
14. Prove that $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (**Hero's Formula**)
15. Find the area of the triangle ABC, given two sides and their included angle
    (i) $a = 200, b = 120, \gamma = 150^0$ (ii) $b = 37, c = 45, \alpha = 30^0 50'$
16. Find the area of the triangle ABC, given one side and two angles
    (i) $b = 25.4, \gamma = 36^0 41', \alpha = 45^0 17'$ (ii) $c = 32, \alpha = 47^0 24', \beta = 70^0 16'$
17. Find the area of the triangle ABC, given three sides
    (i) $a = 18, b = 24, c = 30$ (ii) $a = 524, b = 276, c = 315$
18. The area of triangle is 2437. If $a = 79$ and $c = 97$, then find angle β
19. The area of triangle is 121.34. If $\alpha = 32^0 15', \beta = 65^0 37'$ then find c and angle γ

20. One side of a triangular garden is 30 meter. If its two corner angles are $22^0\frac{1}{2}$ & $112^0\frac{1}{2}$ find the cost of planting the grass at the rate of Rs. 5 per square meter

21. Prove that $R = \frac{abc}{4\Delta}$

22. Prove that $r = (s-a)\tan\frac{\alpha}{2}$ (**Example**) Page 382

23. Prove that $r = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$

24. Show that

(i) $r_1 = 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ (ii) $r_3 = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$

25. Show that

(i) $r_1 = s\tan\frac{\alpha}{2}$ (ii) $r_2 = s\tan\frac{\beta}{2}$

26. Prove that

(i) $rr_1r_2r_3 = \Delta^2$ (ii) $r_1\, r_2r_3 = rs^2$

27. Find R, r, r_1, r_2 and r_3 if measures of the sides of triangle ABC are

(i) $a = 13, b = 14, c = 15$ (ii) $a = 34, b = 20, c = 42$

28. Prove that $\Delta = r^2\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$

29. Show that

(i) $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$ (ii) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

30. Prove that $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$

31. Prove that

(i) $(r_1 + r_2)\tan\frac{\gamma}{2} = c$ (ii) $(r_3 - r)\cot\frac{\gamma}{2} = c$

Long Questions

1. P and Q are two points in line with a tree. If the distance between P and Q be 30 meter and the angles of elevation of the top of the tree at P and Q be 12^0 and 15^0 respectively. Find the height of the tree

2. Two buildings A and B are 100 meter apart. The angle of elevation from the top of the building A to the top of the building B is 20^0. The angle of elevation from the base of the building B to the top of the building A is 50^0. Find the height of the building B

3. Solve the following triangles, using law of tangent and then law of Sine

(i) $a = 36.21, b = 42.09$ & $\gamma = 44^0 29'$ (ii) $b = 61, c = 32$ & $\alpha = 59^0 30'$

4. Prove that laws of cosines

(i) $a^2 = b^2 + c^2 - 2bc\cos\alpha$ (ii) $b^2 = a^2 + c^2 - 2ac\cos\beta$

(iii) $c^2 = a^2 + b^2 - 2ab\cos\gamma$

5. Prove that $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

6. The sides of a triangle are $x^2 + x + 1, 2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120^0

7. Prove that $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$ (**Example**) Page 383

8. Prove that $r_1 + r_2 + r_3 - r = 4R$

9. Prove that in an equilateral triangle

(i) $r : R : r_1 = 1 : 2 : 3$ (ii) $r : R : r_1 : r_2 : r_3 = 1 : 2 : 3 : 3 : 3$

Chapter No.13

Inverse Trigonometric Functions

Short Questions

1. Definitions
- **(i)** Inverse Sine Function
- **(ii)** Inverse Cosine Function
- **(iii)** Inverse Tangent Function
- **(iv)** Inverse Cotangent Function
- **(v)** Inverse Secant Function
- **(vi)** Inverse Cosecant Function

2. Evaluate without using table/ calculator
- **(i)** $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
- **(ii)** $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
- **(iii)** $\cos^{-1}\left(\frac{1}{2}\right)$
- **(iv)** $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
- **(v)** $\cot^{-1}(-1)$
- **(vi)** $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

3. Without using table/ calculate show that
- **(i)** $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$
- **(ii)** $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$

4. Find the value of each expression:
- **(i)** $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (**Ex**) Page 396
- **(ii)** $\cos\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$
- **(iii)** $\sec\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$
- **(iv)** $\tan\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$
- **(v)** $\csc[\tan^{-1}(-1)]$
- **(vi)** $\sin[\tan^{-1}(-1)]$

5. Prove that
- **(i)** $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$
- **(ii)** $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

6. Show that
- **(i)** $\cos(\sin^{-1}x) = \sqrt{1-x^2}$
- **(ii)** $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$
- **(iii)** $\cos(2\sin^{-1}x) = 1-2x^2$
- **(iv)** $\tan^{-1}(-x) = -\tan^{-1}x$
- **(v)** $\sin^{-1}(-x) = -\sin^{-1}(x)$
- **(iv)** $\cos^{-1}(-x) = \pi - \cos^{-1}x$

7. Show that $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$

8. Prove that $2\tan^{-1}A = \tan^{-1}\frac{2A}{1-A^2}$

9. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$

10. Prove that $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$

11. Prove that $\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$

Long Questions

1. Prove that
- **(i)** $\sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$
- **(ii)** $\sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$
- **(iii)** $\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$
- **(iv)** $\cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$

2. Prove the following
- **(i)** $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$
- **(ii)** $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$
- **(iii)** $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$
- **(iv)** $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{15}{17}$

(v) $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$

(vi) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$

(vii) $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

(viii) $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

Chapter No.14

Solutions of Trigonometric Equations

- ## Short Questions

1. Define Trigonometric Equation and reference angle
2. Solve the equation $\sin x = \frac{1}{2}$ (**Example**) Page 401
3. Solve the equation $1 + \cos x = 0$ (**Example**) Page 402
4. Solve the equation $4\cos^2 x - 3 = 0$(**Example**) Page 402
5. Solve $\sin x + \cos x = 0$ (**Example**) Page 403
6. Solve the equation $\sin 2x = \cos x$ (**Example**) Page 404
7. Find the solution of the following equations which lie in $[0, 2\pi]$
   (i) $\sin x = \frac{-\sqrt{3}}{2}$ (ii) $\cot\theta = \frac{1}{\sqrt{3}}$
   (iii) $\csc\theta = 2$ (iv) $\sec x = -2$
8. Solve the following trigonometric equations
   (i) $\tan^2\theta = \frac{1}{3}$ (ii) $\csc^2\theta = \frac{4}{3}$
   (iii) $\sec^2\theta = \frac{4}{3}$ (iv) $\cot^2\theta = \frac{1}{3}$
9. Find the value of θ satisfying the following equations
   (i) $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$ (ii) $\tan^2\theta - \sec\theta - 1 = 0$
   (iii) $2\sin\theta + \cos^2\theta - 1 = 0$ (iv) $2\sin^2\theta - \sin\theta = 0$