2nd Year Mathematics
Great Science Academy Guess Paper By
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For All Punjab Boards (LHR, GRW, SGD, SWL, MTN, RWP, DGK, FSD & BWP)

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Chapter No.01

Functions & Limits

- ## Short Questions

1. Find the Domain and Range of f and g
   a) $f(x) = x^2$
   b) $f(x) = \frac{x}{x^2-4}$
   c) $f(x) = \sqrt{x^2 - 9}$
   d) $f(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ x-1 & \text{if } 1 < x \le 2 \end{cases}$
   f) $g(x) = \begin{cases} 6x+7, & x \le -2 \\ 4-3x, & -2 < x \end{cases}$
   e) $g(x) = \begin{cases} x-1, & x < 3 \\ 2x+1, & 3 \le x \end{cases}$
   g) $g(x) = \frac{x^2-3x+2}{x+1}, x \ne -1$
   h) : $g(x) = |x-3|$
2. Definition of The following
   a) Polynomial Function
   b) Identity function
   c) Hyperbolic Function
   d) Explicit & Implicit Function
   e) Even and Odd Function
   f) Functions
   g) Composition of a Function
   h) Inverse Function
   i) Continuous Function
   j) Discontinuous Function
3. Show that the parametric Equation
   a) $x = a\cos t$ & $y = a\sin t$ represent the equation of circle $x^2 + y^2 = a^2$
   b) $x = at^2; y = 2at$ represent the equation of parabola $y^2 = 4ax$
   c) $x = a\cos\theta; y = b\sin\theta$ represent the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
   d) $x = a\sec\theta; y = b\tan\theta$ represent the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
4. Prove that the following identities
   a) $\cosh^2 x - \sinh^2 x = 1$
   b) $\cosh^2 x + \sinh^2 x = \cosh 2x$
   c) $\sinh 2x = 2\sinh x\cosh x$
   d) $\text{sech}^2 = 1 - \tan h^2 x$
   e) $\text{csch}^2 = \cot h^2 x - 1$
5. Determine the function f is even or odd if
   a) $(x) = \sin x + \cos x$
   b) $f(x) = \frac{3x}{x^2+1}$
   c) $f(x) = (x+2)^2$
   d) $f(x) = \frac{x-1}{x+1}, x \ne -1$
   e) $f(x) = x^{2/3} + 6$
   f) $f(x) = x\sqrt{x^2+5}$
6. Find $f(x-1)$ & $f(x^2+4)$ if
   a) $f(x) = x^2 - x$
   b) $f(x) = \sqrt{x+4}$
7. Find $\frac{f(a+h)-f(a)}{h}$ and simplify
   a) $f(x) = \cos x$
   b) $f(x) \sin x$
8. Express the following

- **a)** The perimeter P of a square as a function of its area A
- **b)** The area A of a circle as a function of its circumference C
- **c)** The volume V of a cube as a function of the area A of its base

9. Find $f^{-1}(x)$ if $f(x) = -2x + 8$

10. Find $fog(x); gog(x); fof(x); gof(x)$

- **a)** $f(x) = 2x + 1$
- **b)** $g(x) = (x^2 + 1)^2$

11. Let the real valued function 'f' and 'g' be defined by $fg(x)$ $f^2(x)$ & $g^2(x)$

- **a)** $f(x) = 2x + 1$
- **b)** $g(x) = x^2 - 1$

12. Without finding the inverse, state the domain and range of f^{-1} if

- **a)** $f(x) = \sqrt{x+2}$
- **b)** $f(x) = \frac{1}{x+3}, x \neq -3$
- **c)** $f(x) = \frac{x-1}{x-4}, x \neq 4$
- **d)** $f(x) = (x-5)^2, x \geq 5$

13. Evaluate $\lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$

14. Evaluate

- **a)** $\lim_{x\to 1}\frac{x^2-1}{x^2-x}$
- **b)** $\lim_{x\to 3}\frac{x-3}{\sqrt{x}-\sqrt{3}}$

15. Evaluate

- **a)** $\lim_{x\to +\infty}\frac{5x^4-10x^2+1}{-3x^3+10x^2+50}$
- **b)** $\lim_{x\to -\infty}\frac{4x^4-5x^3}{3x^5+2x^2+1}$

16. Evaluate

- **a)** $\lim_{x\to -\infty}\frac{2-3x}{\sqrt{3+4x^2}}$
- **b)** $\lim_{x\to +\infty}\frac{2-3x}{\sqrt{3+4x^2}}$

17. Evaluate

- **a)** $\lim_{\theta\to 0}\frac{\sin 7\theta}{\theta}$
- **b)** $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}$

18. Evaluate limit by using the theorem of limits

- **a)** $\lim_{x\to 3}\sqrt{x^2+x+4}$
- **b)** $\lim_{x\to 2}x\sqrt{x^2-4}$
- **c)** $\lim_{x\to 2}\left(\sqrt{x^3+1}-\sqrt{x^2+5}\right)$
- **d)** $\lim_{x\to -2}\frac{2x^3+5x}{3x-2}$

19. Evaluate each limit by using algebraic techniques

- **a)** $\lim_{x\to -1}\frac{x^3-x}{x+1}$
- **b)** $\lim_{x\to 2}\frac{x^3-8}{x^2+x-6}$
- **c)** $\lim_{x\to 2}\frac{\sqrt{x}-\sqrt{2}}{x-2}$
- **d)** $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$
- **e)** $\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}$

20. Evaluate the limits

- **a)** $\lim_{x\to 0}\frac{\sin x^0}{x}$
- **b)** $\lim_{x\to \pi}\frac{\sin x}{\pi-x}$
- **c)** $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$
- **d)** $\lim_{x\to 0}\frac{x}{\tan x}$
- **e)** $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

21. Express each limit in term of e

- **a)** $\lim_{n\to +\infty}\left(1-\frac{1}{n}\right)^n$
- **b)** $\lim_{n\to +\infty}\left(1+\frac{4}{n}\right)^n$
- **c)** $\lim_{h\to 0}(1-2h)^{1/h}$
- **d)** $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$

22. For $f(x) = 3x^2 - 5x + 4$, discuss continuity at $x = 1$

23. Determine the left & right hand limit and find limit

- **a)** $f(x) = |x-5|, c = 5$
- **b)** $f(x) = \frac{x^2-9}{x-3}; c = -3$

24. If $f(x) = \begin{cases} x+2; x \leq -1 \\ c+2; x > -1 \end{cases}$, find "c" so that $\lim_{x\to -1} f(x)$ exist

25. Find the value of "m" so that function f is continuous at $x = 3$: $f(x) = \begin{cases} mx \text{ if } x < 3 \\ x^2 \text{ if } x \geq 3 \end{cases}$

Long Questions

1. Given $f(x) = x^3 - ax^2 + bx + 1$ if $f(2) = -3$ & $f(-1) = 0$. Find the values of "a" and "b"

2. Verify that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$ if

- **a)** $f(x) = 3x^3 + 7$
- **b)** $f(x) = (-x+9)^3$
- **c)** $f(x) = \frac{2x+1}{x-1}, x > 1$

3. Prove that $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$, where n is an integer and $a > 0$

4. Prove that

a) $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e$ b) $\lim_{x\to 0}(1+x)^{1/x} = e$

5. Prove that $\lim_{x\to 0}\frac{a^x-1}{x} = \log_e a = \ln a$

6. If θ is measured in radian, then $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

7. Evaluate $\lim_{x\to a}\frac{x^n-a^n}{x^m-a^m}$

8. Evaluate

a) $\lim_{x\to 0}\frac{\sec x-\cos x}{x}$ b) $\lim_{\theta\to 0}\frac{1-\cos p\theta}{1-\cos q\theta}$ c) $\lim_{\theta\to 0}\frac{\tan\theta-\sin\theta}{\sin^3\theta}$

9. Express each limit in term of e

a) $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}, x<0$ b) $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}, x>0$

10. Discuss the continuity of the function f(x) and g(x) at x = 3

a) $f(x) = \begin{cases}\frac{x^2-9}{x-3} & \text{if } x\neq 3\\ 6 & \text{if } x=3\end{cases}$ b) $g(x) = \frac{x^2-9}{x-3}$ if $x \neq 3$

11. Discuss the continuity of

a) $(x) = \begin{cases}2x+5 \text{ if } x\le 2\\ 4x+1 \text{ if } x>2\end{cases}, c=2$ b) $f(x) = \begin{cases}3x-1 & \text{if } x<1\\ 4 & \text{if } x=1\\ 2x & \text{if } x>1\end{cases}, c=1$

12. Find the value of “m” and “n” so that given function f is continuous at x = 3:

$$f(x) = \begin{cases}mx & \text{if } x<3\\ n & \text{if } x=3\\ -2x+9 & \text{if } x>3\end{cases}$$

13. If $f(x) = \begin{cases}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$ Find value of k so that f is continuous at x = 2

Chapter No.02

Differentiation

- ## Short Questions

1. Find the derivative of the functions by definition $f(x) = c$ & $f(x) = x^2$
2. Finding $f'(x)$ from definition of derivative
3. If $y = \frac{1}{x^2}$, the find $\frac{dy}{dx}$ at $x = -1$ by ab – initio method
4. Find by definition, the derivative w.r.t. “x” of the following function defined as: x^3+2x+3
5. Find the derivative of w.r.t. ”x”

a) $y = \frac{3}{4}x^4+\frac{2}{3}x^3+\frac{1}{2}x^2+2x+5$ b) $y = (x^2+5)(x^3+7)$

c) $y = (2\sqrt{x}+2)(x-\sqrt{x})$

6. Find $\frac{dy}{dx}$ if

a) $y = \frac{(\sqrt{x}+1)(x^{3/2}-1)}{x^{\frac{1}{2}}-1}$ b) $y = \frac{(\sqrt{x}+1)(x^{3/2}-1)}{x^{3/2}-x^{1/2}}$ c) $y = \frac{2x^3-3x^2+5}{x^2+1}$

7. Differentiate w. r. t. “x”

i $y = x^4+2x^3+x^2$ ii $y = x^{-3}+2x^{-3/2}+3$

iii $y = \frac{2x-3}{2x+1}$ iv $y = \frac{a+x}{a-x}$

v $y = (x-5)(3-x)$ vi $y = \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$

vii $y = \frac{x^2+1}{x^2-3}$

viii If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$

8. Find $\frac{dy}{dx}$ if $y = \left(1+2\sqrt{x}\right)^3 . x^{3/2}$

9. Find $\frac{dy}{dx}$ if

a) $x = at^2$ & $y = 2at$　　**b)** $x = 1 - t^2$ & $y = 3t^2 - 2t^3$

10. Find $\frac{dy}{dx}$ if

a) $x^2 + y^2 = 4$　　**b)** $y^2 + x^2 - 4x = 5$　　**c)** $y^2 - xy - x^2 + 4 = 0$

11. Differentiate $x^2 + \frac{1}{x^2}$ w.r.t $x - \frac{1}{x}$

12. Find $\frac{dy}{dx}$ by making suitable substitutions in the following function defined as:

a) $y = \sqrt{x + \sqrt{x}}$　　**b)** $y = (3x^2 - 2x + 7)^6$

13. Find $\frac{dy}{dx}$ if:

a) $3x + 4y = 7 = 0$　　**b)** $xy + y^2 = 2$

c) $x^2 - 4xy - 5y = 0$　　**d)** $4x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

14. Find $\frac{dy}{dx}$ if $x = \theta + \frac{1}{\theta}$ and $y = \theta + 1$

15. Find $\frac{dy}{dx}$ if

a) $y = \sec x$　　**b)** $y = \operatorname{cosec} x$　　**c)** $y = \cot x$

16. Differentiate $\sin^3 x$ w.r.t. $\cos^2 x$

17. Prove that

a) $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$　　**b)** $\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$

c) $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{x^2+1}$　　**d)** $\frac{d}{dx}[\cot^{-1} x] = \frac{-1}{x^2+1}$

18. Differentiate the following w.r.t. the variable involved

a) $y = x^2 \sec 4x$　　**b)** $y = \tan^3\theta . \sec^2\theta$

c) $y = (\sin 2\theta - \cos 3\theta)^2$　　**d)** $y = \cos\sqrt{x} + \sqrt{\sin x}$

19. Find $\frac{dy}{dx}$ if

a) $y = x\cos y$　　**b)** $x = y\sin y$

20. Differentiate

a) $\sin x$ w.r.t. $\cot x$　　**b)** $\sin^2 x$ w.r.t. $\cos^4 x$

21. If $\tan y\,(1 + \tan x) = 1 - \tan x$, show that $\frac{dy}{dx} = -1$

22. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x} + \cdots\infty}}$, prove that $(2y-1)\frac{dy}{dx} = \sec^2 x$

23. Differentiate w.r.t. "x"

a) $y = \cos^{-1}\frac{x}{a}$　　**b)** $y = \cot^{-1}\frac{x}{a}$

c) $y = \frac{1}{a}\sin^{-1}\frac{a}{x}$　　**d)** $y = \sin^{-1}\sqrt{1-x^2}$

24. If $y = \tan(p\tan^{-1}x)$, show that $(1+x^2)y_1 - p(1+y^2) = 0$

25. Find $\frac{dy}{dx}$

a) $y = e^{x^2+1}$　　**b)** $y = a^{\sqrt{x}}$　　**c)** $y = a^x$

26. Find $\frac{dy}{dx}$ if

a) $y = \log_{10}(ax^2 + bx + c)$　　**b)** $y = \ln(x^2 + 2x)$

27. Differentiate $(\ln x)^x$ w.r.t. "x"

28. Prove that

a) $\frac{d}{dx}(\sinh hx) = \cos hx$ b) $\frac{d}{dx}(\cos hx) = \sin hx$ c) $\frac{d}{dx}(\tan hx) = \sec h^2 x$

29. Find $\frac{dy}{dx}$ if

a) $y = \sinh 2x$ b) $y = \tanh(x^2)$
c) $y = \sinh^{-1}(ax + b)$ d) $y = \cosh^{-1}(\sec x)$

30. Find $\frac{dy}{dx}$ if

a) $y = e^{\sqrt{x}-1}$ b) $y = x^3 . e^{1/x}$ c) $y = e^x\,(1 + \ln x)$
d) $y = \ln(e^x + e^{-x})$ e) $y = \sqrt{\ln(e^{2x} + e^{-2x})}$ f) $y = \ln\left(\sqrt{e^{2x} + e^{-2x}}\right)$
g) $y = x^2 \ln\sqrt{x}$ h) $y = x\sqrt{\ln x}$ i) $y = \frac{x}{\ln x}$
j) $y = x^2 \ln\frac{1}{x}$ k) $y = \ln(x + \sqrt{x^2 + 1})$ l) $y = \ln(9 - x^2)$
m) $y = e^{-2x}\sin 2x$ n) $y = e^{-x}(x^3 + 2x^2 + 1)$ o) $y = x\,e^{\sin x}$
p) $y = \tanh^{-1}(\sin x)$ q) $y = \sinh^{-1}(x^3)$ r) $y = \ln(\tanh x)$

31. Find the higher derivative of the polynomial $f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 + \frac{1}{4}x^2 + 2x + 7$

32. Find y_2 if

a) $y = 2x^5 - 3x^4 + 4x^3 + x - 2$ b) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$
c) $y = (2x + 5)^{3/2}$ d) $y = \ln\left(\frac{2x+3}{3x+2}\right)$
e) $y = x^2 . e^{-x}$ f) $x^2 + y^2 = a^2$
g) $x^3 - y^3 = a^3$ h) $x = at^2; y = bt^4$

33. Find y_4 if (a) $y = \sin 3x$ (b) $y = \cos^3 x$ (c) $y = \text{in}\,(x^2 - 9)$

34. Find the Maclaurin Series for the following functions

a) $f(x) = \sin x$ b) $f(x) = \frac{1}{x+1}$ c) $f(x) = a^x$

35. Apply the Maclaurin Series expansion to prove that

a) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
b) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
c $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \cdots$

36. Prove that

a) $e^{x+h} = e^x\left\{1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \cdots\right\}$ b) $2^{x+h} = 2^x\left\{1 + (\ln 2)\,h + \frac{(\ln 2)^2 h^2}{2!} + \frac{(\ln 2)^3 h^3}{3!} + \cdots\right\}$

37. Definition of the following

a) Increasing & Decreasing Functions b) Stationary Point c) Critical Point
d) Point of Inflection e) 1st derivative test f) 2nd derivative test

38. Determine the interval in which f is increasing or decreasing for the domain mentioned in each case

a) $f(x) = \sin x; x \in (-\pi, \pi)$ b) $f(x) = 4 - x^2; x \in (-2,2)$
c) $f(x) = \cos x; x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ d) $f(x) = x^2 + 3x + 2; x \in (-4,1)$

39. Find the extreme values for the following functions defined as:

a) $f(x) = 1 - x^3$ b) $f(x) = 1 + x^3$
c) $f(x) = x^2 - x - 2$ d) $f(x) = 5x^2 - 6x + 2$
e) $f(x) = 3x^2$ f) $f(x) = x^4 - 4x^2$

Long Questions

1. Find by definitions, the derivative w.r.t. "x" of the following functions defined
   a) $\frac{1}{\sqrt{x}}$ b) $\frac{1}{x-a}$ c) $(x+4)^{\frac{1}{3}}$ d) $x^m; m \in N$ e) $\frac{1}{x^m}; m \in N$
   f) x^{-100} g) $\sqrt{x+2}$ h) $\frac{1}{\sqrt{x+a}}$ i) $\frac{1}{(az-b)^7}$ j) $(3t+2)^{-2}$
2. Differentiate w.r.t. "x"
   a) $\sqrt{\frac{1+x}{1-x}}$ b) $\sqrt{\frac{a-x}{a+x}}$ c) $\sqrt{\frac{x^2+1}{x^2-1}}$
   d) $x\sqrt{\frac{a+x}{a-x}}$ f) $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$ g) $\frac{2x-1}{\sqrt{x^2+1}}$
3. If $y=\sqrt{x}-\frac{1}{\sqrt{x}}$, show that $2y\frac{dy}{dx}+y=2\sqrt{x}$
4. Find $\frac{dy}{dx}$ if $\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}; x \neq 0$
5. Find $\frac{dy}{dx}$ if $y=x^n$ where $n=\frac{p}{q}, q \neq 0$
6. Find $\frac{dy}{dx}$ by making suitable substitutions function defined as: $y=x\sqrt{\frac{a+x}{a-x}}$
7. Find $\frac{dy}{dx}$ if
   a) $x\sqrt{1+y}+y\sqrt{1+x}=0$ b) $y(x^2-1)=x\sqrt{x^2+4}$
8. Prove that $y\frac{dy}{dx}+x=0$ if $x=\frac{1-t^2}{1+t^2}, y=\frac{2t}{1+t^2}$
9. Differentiate
   a) $\frac{x^2+1}{x^2-1}$ w.r.t. $\frac{x-1}{x+1}$ b) $\frac{ax+b}{cx+d}$ w.r.t. $\frac{ax^2+b}{ax^2+d}$
10. Differentiate ab – initio w.r.t. "x"
    a) $y=\sin\sqrt{x}$ b) $y=\cot^2 x$
    c) $y=\cos\sqrt{x}$ d) $y=\cos x^2$
    e) $y=\sqrt{\tan x}$ f) $y=\tan^2 x$
11. Find $\frac{dy}{dx}$ if $y=x\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}$
12. If $y=\tan\left(2\tan^{-1}\frac{x}{2}\right)$, show that $\frac{dy}{dx}=\frac{4(1+y^2)}{(4+x^2)}$
13. If $x=a\cos^3\theta; y=b\sin^3\theta$, show that $a\frac{dy}{dx}+b\tan\theta=0$
14. Find $\frac{dy}{dx}$ if $x=a(\cos t+\sin t); y=a(\sin t-t\cos t)$
15. Show that $\frac{dy}{dx}=\frac{y}{x}$ if $\frac{y}{x}=\tan^{-1}\frac{x}{y}$
16. Find $\frac{d^3y}{dx^3}$ if $y=\ln(x+\sqrt{x^2+a^2})$
17. Find $\frac{d^2y}{dx^2}$ if $y^3+3ax^2+x^3=0$
18. If $x=a(\theta-\sin\theta); y=a(\theta+\sin\theta)$. Then show that $y^2\frac{d^2y}{dx^2}+a=0$
19. If $y=a^{-ax}$, then show that $\frac{d^3y}{dx^3}+a^3y=0$
20. If $y=\sin^{-1}\frac{x}{a}$, then show that $y_2=x(a^2-x^2)^{-3/2}$
21. If $x=\sin\theta; y=\sin m\theta$, show that $(1-x^2)y_2-xy_1+m^2y=0$
22. If $y=e^x\sin x$, show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$
23. If $y=e^{ax}\sin bx$, show that $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$

24. If $y=(\cos^{-1}x)^2$, prove that $(1-x^2)y_2-xy_1-2=0$

25. If $y=a\cos(\ln x)+b\sin(\ln x)$, prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

26. Use the Taylor series expansion to find the value of $\sin 31^0$

27. Show that $\cos(x+h)=\cos x-h\sin x-\frac{h^2}{2!}\cos x+\frac{h^2}{3!}\sin x+\cdots$ and evaluate $\cos 61^0$

28. Find the maximum and minimum values of the function defined by the following equation occurring in the interval $[0,2\pi]$ such that $f(x)=\sin x+\cos x$

29. Show that $y=\frac{\ln x}{x}$ has maximum value at $x=e$

30. Show that $y=x^x$ has maximum value at $x=\frac{1}{e}$

Chapter No.03

Integration

Short Questions

1. Find δy and dy in the following cases: **(Exercise 3.1)**

- a) $y=x^2$, when $x=2$ and $dx=0.01$ (Example)(Page 121)
- b) $y=x^2-1$ when x changes from 3 to 3.02
- c) $y=x^2-2x$ when x changes from 2 to 1.8
- d) $y=\sqrt{x}$ when x changes from 4 to 4.41

2. Using the differentials find $\frac{dy}{dx}$ & $\frac{dx}{dy}$ in the following equations

- a) $\frac{y}{x}-\ln x=\ln c$ (Example) Page 121
- b) $xy+x=4$
- c) $x^2+2y^2=16$
- d) $xy-\ln x=c$
- e) $x^4+y^2=xy^2$

3. Use differentials to approximate the values of

- a) $\sqrt{17}$ (Example)Page 122
- b) $\sqrt[3]{8.6}$ (Example)Page 122
- c) $\sin 46^0$(Example)Page 122
- d) $\sqrt[3]{17}$
- e) $31^{1/5}$
- f) $\sin 61^0$ & $\cos 29^0$

4. Evaluate (Examples) (Page 128)

- a) $\int x\sqrt{x^2-1}\,dx$
- b) $\int\frac{dx}{\sqrt{x}(\sqrt{x}+1)}$
- c) $\int\frac{\sin x+\cos^3 x}{\cos^2 x\sin x}dx$
- d) $\int\frac{3-\cos 2x}{1+\cos 2x}dx$
- e) $\int\frac{x}{x+2}dx$

5. Evaluate **(Exercise 3.2)**

- a) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$
- b) $\int(2x+3)^{1/2}dx$
- c) $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$
- d) $\int\frac{3x+2}{\sqrt{x}}dx$
- e) $\int\frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}d\theta$
- f) $\int\frac{(1-\sqrt{x})^2}{\sqrt{x}}dx$
- g) $\int\frac{e^{2x}+e^x}{e^x}dx$
- h) $\int(\sqrt{x}+1)^2dx$

6. Evaluate

- a) $\int\frac{1-x^2}{1+x^2}dx$
- b) $\int(a-2x)^{3/2}dx$
- c) $\int\frac{(1+e^x)^3}{e^x}dx$
- d) $\int\sqrt{1-\cos 2x}\,dx$
- e) $\int\ln x\times\frac{1}{x}dx$
- f) $\int\frac{1}{1+\cos x}dx$
- g) $\int\sin^2 dx$
- h) $\int\frac{ax+b}{ax^2+bx+c}dx$
- i) $\int\frac{\cos 2x-1}{1+\cos 2x}dx$
- j) $\int\cos 3x.\sin 2x\,dx$
- k) $\int\tan^2 dx$

7. Evaluate (Examples)(Page131 – 137)

- a) $\int\frac{a\,dt}{2\sqrt{at+b}}$
- b) $\int x\sqrt{x-a}dx$
- c) $\int\frac{\cot\sqrt{x}}{\sqrt{x}}dx$
- d) $\int\operatorname{cosec} dx$
- e) $\int\sec dx$
- f) $\int\cos^3 x\sqrt{\sin x}\,dx$
- g) $\int\sqrt{1+\sin x}\,dx$
- h) $\int a^{x^2}dx$
- i) $\int\frac{dx}{x(\ln 2x)^3}$
- j) $\int\frac{dx}{x\sqrt{x^2-a^2}}$
- k) $\int\frac{dx}{\sqrt{2x+x^2}}$

8. Evaluate the following integrals **(Exercise 3.3)**

- a) $\int\frac{-2x}{\sqrt{4-x^2}}dx$
- b) $\int\frac{dx}{x^2+4x+13}$
- c) $\int\frac{x^2}{4+x^2}dx$
- d) $\int\frac{1}{x\ln x}dx$
- e) $\int\frac{e^x}{e^x+3}dx$
- f) $\int\frac{x+b}{(x^2+2bx+c)^{1/2}}dx$
- g) $\int\frac{\sec^2 x}{\sqrt{\tan x}}dx$
- h) $\int\frac{dx}{(1+x^2)^{3/2}}$

i) $\int \frac{dx}{(1+x^2)\tan^{-1}x}$ j) $\int \frac{\sin\theta}{1+\cos^2\theta}d\theta$ k) $\int \frac{ax}{\sqrt{a^2-x^4}}dx$ l) $\int \frac{\cos x}{\sin x \ln \sin x}dx$

m) $\int \cos x\left(\frac{\ln \sin x}{\sin x}\right)dx$ n) $\int \left[\cos\left(\sqrt{x}-\frac{x}{2}\right)\times\left(\frac{1}{\sqrt{x}}-1\right)\right]dx$ o) $\int \frac{\sqrt{2}}{\sin x+\cos x}dx$ p) $\int \frac{dx}{\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x}$

9. Evaluate (Example)(Page 138 – 144)

a) $\int x\,e^x\,dx$ b) $\int x\tan^2 x\,dx$ c) $\int x^5\ln x dx$ d) $\int \ln\left(x+\sqrt{x^2+1}\right)dx$

e) $\int x^2.ae^{ax}dx$

10. Evaluate the following integrals by part **(Exercise 3.4)**

a) $\int x\sin x dx$ b) $\int \ln x\,dx$ c) $\int \tan^{-1}x\,dx$ d) $\int x^2\sin x\,dx$

e) $\int x^3\tan^{-1}x\,dx$ f) $\int \sin^{-1}x\,dx$ g) $\int e^x\sin x\cos x\,dx$ h) $\int x\sin x\cos x dx$

i) $\int x\cos^2 dx$ j) $\int (\ln x)^2\,dx$ k) $\int x\sin^2 dx$ l) $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

m) $\int \ln(\tan).\sec^2 x\,dx$

11. Evaluate the following integral

a) $\int \tan^4 dx$ b) $\int \sec^4 dx$ c) $\int \tan^3 x\sec x dx$ d) $\int x^3 e^{5x}dx$

e) $\int e^{-x}\sin 2x\,dx$ f) $\int e^{2x}\cos 3x\,dx$ g) $\int \mathrm{cosec}^3 x\,dx$

12. Evaluate the following integrals

a) $\int e^x\left(\frac{1}{x}+\ln x\right)dx$ b) $\int e^x(\cos x+\sin x)dx$ c) $\int e^{ax}\left[a\sec^{-1}x+\frac{1}{x\sqrt{x^2-1}}\right]dx$

d) $\int e^{3x}\left(\frac{3\sin x-\cos x}{\sin^2 x}\right)dx$ e) $\int e^{2x}[-\sin x+2\cos x]dx$ f) $\int \frac{e^{m\tan^{-1}x}}{(1+x^2)}dx$

g) $\int \frac{2x}{1-\sin x}dx$ h) $\int \left(\frac{1-\sin x}{1-\cos x}\right)dx$ i) $\int \frac{e^x(1+x)}{(2+x)^2}dx$

13. Evaluate (Example)(Page 145 – 150)

a) $\int \frac{-x+6}{2x^2-7x+6}dx$ b) $\int \frac{2x^3-9x^2+12x}{2x^2-7x+6}dx$ c) $\int \frac{2a}{x^2-a^2}dx$

d) $\int \frac{2a}{a^2-x^2}dx$

14. Evaluate (Examples)(Pages 157 – 162)

a) $\int_{-1}^{3}(x^3+3x^2)dx$ b) $\int_1^2 \frac{x^2+1}{x+1}dx$ c) $\int_0^{\pi/4}\sec x(\sec x+\tan x)dx$

d) $\int_0^{\pi/4}\frac{dx}{1-\sin x}$ e) $\int_{-1}^{2}(x+|x|)dx$ f) $\int_0^{\pi/6}x\cos x dx$

15. Evaluate the definite integrals **(Exercise 3.6)**

a) $\int_1^2(x^2+1)dx$ b) $\int_{-1}^{1}\left(x^{1/3}+1\right)dx$ c) $\int_{-2}^{0}\frac{dx}{(2x-1)^2}$

d) $\int_1^{\sqrt{5}}\sqrt{(2t-1)^3}dt$ e) $\int_2^{\sqrt{5}}x\sqrt{x^2-1}dx$ f) $\int_1^2\frac{x}{x+2}dx$

g) $\int_2^3\left(x-\frac{1}{x}\right)^2dx$ h) $\int_{-1}^{1}\left(x+\frac{1}{2}\right)\sqrt{x^2+x+1}dx$ i) $\int_0^3\frac{1}{x^2+9}dx$

j) $\int_{\pi/6}^{\pi/3}\cos t\,dt$ l) $\int_1^2\left(x+\frac{1}{x}\right)^{1/2}\left(1-\frac{1}{x^2}\right)dx$ m) $\int_1^2\ln x\,dx$

n) $\int_0^2\left(e^{x/2}-e^{-x/2}\right)dx$ o) $\int_0^{\pi/3}\cos^2\theta\sin\theta\,d\theta$ p) $\int_0^{\pi/4}\frac{\sec\theta}{\sin\theta+\cos\theta}d\theta$

q) $\int_{-1}^{5}|x-3|dx$ r) $\int_{1/8}^{1}\frac{\left(x^{1/3}+2\right)^2}{x^{2/3}}dx$ s) $\int_1^3\frac{x^2-2}{x+1}dx$

t) $\int_2^3\frac{3x^2-2x+1}{(x-1)(x^2+1)}dx$ u) $\int_0^{\pi/4}\frac{1}{1+\sin x}dx$ v) $\int_0^1\frac{3x}{\sqrt{4-3x}}dx$

16. Find the area bounded by (Example)(Pages 164 – 167)

a) The curve $y=4-x^2$ and the x – axis

b) The curve $y=x^3+3x^2$ and the x – axis

c) The curve $y=x(x^2-4)$ and the x – axis

d) The curve $y=x^3-2x^2+1$ and the x – axis in 1st quadrant

e) The curve $y^2=4-x$ and the x – axis in the 1st quadrant from $x=0$ to $x=3$

17. Find the area bounded by **(Exercise 3.7)**

a) The curve $y=3\sqrt{x}$ and the above x – axis between $x=1$ to $x=4$

b) The curve $y=\cos x$ and the above x – axis between $x=\frac{-\pi}{2}$ to $x=\frac{\pi}{2}$

c) The curve $y = x^2 - 4x$ and the x – axis
d) The curve $y = \cos\frac{x}{2}$ from $x = -\pi$ and $x = \pi$
e) The curve $y = \sin 2x$ and the above x – axis between $x = \frac{-\pi}{2}$ to $x = \frac{\pi}{2}$

18. Solve the differential equation
a) $(x - 1)dx + ydy = 0$
b) $x^2(2y + 1)\frac{dy}{dx} - 1 = 0$
c) $\frac{1}{x}\frac{dy}{dx} - 2y = 0$
d) $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
e) $(\sin y + y\cos y)dy = [x(2\ln x + 1)]dx$
f) $\frac{dy}{dx} = \frac{3}{4}x^2 + x - 3$, if $x = 2; y = 0$

19. Check that equations written against the differential equation its solution **(Exercise 3.8)**
a) $x\frac{dy}{dx} = 1 + y; y = cx - 1$
b) $x^2(2y + 1)\frac{dy}{dx} - 1 = 0; y^2 + y = c - \frac{1}{x}$
c) $y\frac{dy}{dx} - e^{2x} = 1; y^2 = e^{2x} + 2x + c$
d) $\frac{1}{x}\frac{dy}{dx} - 2y = 0; y = ce^{x^2}$
e) $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}; y = \tan(e^x + c)$

20. Solve the following differential equations:
a) $\frac{dy}{dx} = -y$
b) $ydx + xdy = 0$
c) $\frac{dy}{dx} = \frac{1-x}{y}$
d) $\frac{dy}{dx} = \frac{y}{x^2}$
e) $\sin y \operatorname{cosec} x\frac{dy}{dx} = 1$
f) $\frac{1}{x}\frac{dy}{dx} = \frac{1}{2}(1 + y^2)$
g) $2x^2y\frac{dy}{dx} = x^2 - 1$
h) $(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$
i) $\sec^2 x\tan y\, dx + \sec^2 y\tan x dy = 0$
j) $1 + \cos x\tan y\frac{dy}{dx} = 0$
k) $\sec x + \tan y\,\frac{dy}{dx} = 0$
l) $(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}$

Long Questions

1. Show that
a) $\int\frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + c$
b) $\int\sqrt{a^2 - x^2}\,dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + c$

2. Prove that $\int e^{ax}\cos bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

3. Evaluate
a) $\int\sqrt{a^2 + x^2}dx$
b) $\int\sin^4 x\,dx$
c) $\int\frac{e^x(1+\sin x)}{1+\cos x}dx$

4. Evaluate by using integration by part
a) $\int e^x\sin 2x\cos x\,dx$
b) $\int\frac{2x}{1-\sin x}dx$
c) $\int\left(\frac{1-\sin x}{1-\cos x}\right)e^x dx$
d)

5. Prove that $\int e^{ax}\sin bx\,dx = \frac{1}{\sqrt{a^2+b^2}}e^{ax}\sin\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

6. Evaluate by using integration by part
a) $\int\sqrt{a^2 - x^2}\,dx$
b) $\int\sqrt{4 - 5x^2}\,dx$
c) $\int\sqrt{3 - 4x^2}\,dx$
d) $\int\sqrt{x^2 + 4}\,dx$
e) $\int\sqrt{x^2 - a^2}\,dx$

7. Evaluate
a) $\int\frac{e^x(x^2+1)}{(x+1)^2}dx$
b) $\int\frac{1}{x^3-1}dx$
c) $\int\frac{2x}{x^6-1}dx$
d) $\int\frac{3}{x(x^3-1)}dx$

8. Evaluate the integrals
a) $\int\frac{3x+1}{x^2-x-6}dx$
b) $\int\frac{5x+8}{(x+3)(2x-1)}dx$
c) $\int\frac{(a-b)x}{(x-a)(x-b)}dx$
d) $\int\frac{2x}{x^2-a^2}dx$
e) $\int\frac{3-x}{1-x-6x^2}dx$
f) $\int\frac{x-2}{(x+3)(x^2+1)}dx$

g) $\int \frac{12}{x^3+8}dx$ h) $\int \frac{9x+6}{(x^3-8)}dx$ i) $\int \frac{6a^2}{(x^2+a^2)(x^2+4a^2)}dx$

9. Evaluate the definite integral

a) $\int_0^{\sqrt{3}} \frac{x^3+9x+1}{x^2+9}dx$ b) $\int_{1/2}^{\sqrt{3}/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ c) $\int_1^e x\ln x\,dx$

10. Evaluate the definite integral

a) $\int_0^{\pi/4} \frac{\cos\theta+\sin\theta}{2\cos^2\theta}d\theta$ b) $\int_{\pi/6}^{\pi/4} \cos^2\theta\cot^2\theta\,d\theta$ c) $\int_0^{\pi/4} \cos^4 t\,dt$

d) $\int_0^{\pi/4}(1+\cos^2\theta)\tan^2\theta\,d\theta$ e) $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin x(2+\sin x)}dx$ f) $\int_0^{\pi/2} \frac{\sin x}{(1+\cos x)(2+\cos x)}dx$

11. Find the area between the x – axis and the curve $y=\sqrt{2ax-x^2}$ when $a>0$

12. Solve the differential equation

a $2e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$ where $0<y>\frac{\pi}{2}$

b) $\left(y-x\frac{dy}{dx}\right)=2\left(y^2+\frac{dy}{dx}\right)$

13. Find the general solution of the equation $\frac{dy}{dx}-x=xy^2$ also find the particular solution if $y=1$ when $x=0$

14. Solve the differential equation $\frac{dx}{dt}=2x$ given that $x=4$ when $t=0$

Chapter No.04

Introduction to Analytical Geometry

• Short Questions

1. Find the coordinate of the point that divides the join of $A(-6,3),B(5,-2)$ in the ratio $2:3$
   a) Internally b) Externally
2. The point $C(-5,3)$ is the centre of a circle and $P(7,-2)$ lies on the circle what is the radius of the circle
3. Show that
   a) The point $A(0,2),B(\sqrt{3},-1)$ and $C(0,-2)$ are the vertices of a right triangle.
   b) The point $A(3,1),B(-2,-3)$ and $C(2,2)$ are the vertices of an isosceles triangle
4. Find "h" such that the point $A(\sqrt{3},-1),B(0,2)$ and $C(h,-2)$ are vertices of right triangle with right angle at the vertex A
5. Find "h" such that $A(-1,h),B(3,2)$ and $C(7,3)$ are collinear
6. The point $A(-5,-2)$ & $B(5,-4)$ are end of the diameter of a circle. Find centre and radius of the circle
7. Find the points trisecting the join of $A(-1,4)$ & $B(6,2)$
8. Find the point three – fifth of the way along the line segment from $A(-5,8)$ to $B(5,3)$
9. Find the point P on the join of $A(1,4)$ & $B(5,6)$ that is twice as far from A as B is from A & lies
   a) On the same sides of A as B does b) On the opposite side of A as B does
10. Find the points that divide the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ into four equal parts
11. Definition of
    a) Translation of Axes b) Rotation of Axes
12. Find the XY – coordinates, if xy – coordinates are given
    a) $P(3,2);O'(1,3)$ b) $P(-2,6);O'(-3,2)$
    c) $P(-6,-8);O'(-4,-6)$ d) $P\left(\frac{3}{2},\frac{5}{2}\right);O'\left(-\frac{1}{2},\frac{7}{2}\right)$
13. Find xy – coordinates, if XY – coordinates are given
    a) $P(8,10);O'(3,4)$ b) $P(-5,-3);O'(-2,-6)$
    c) $P\left(-\frac{3}{2},-\frac{7}{2}\right);O'\left(\frac{1}{4},-\frac{1}{6}\right)$ d) $P(4,-3);O'(-2,3)$

14. Find the XY – coordinates, if xy – coordinates are given
- **a)** $P(5,3); \theta = 45^0$
- **b)** $P(3,-7); \theta = 30^0$
- **c)** $P(11,-15); \theta = 45^0$
- **d)** $P(15,10); \theta = \tan^{-1}\frac{1}{3}$

15. Find the XY – coordinates, if xy – coordinates are given
- **a)** $P(-5,3); \theta = 30^0$
- **b)** $P(-7\sqrt{2}, 5\sqrt{2}); \theta = 45^0$

16. By considering the area of the region bounded by the triangle with vertices $A(1,4); B(2,-3)$ & $C(6,-10)$

17. Find the area of the region bounded by the triangle with vertices $(a, b+c); (a, b-c); (-a, c)$

18. Find the slope and inclination of the line joining the points $(3,-2); (2,7)$

19. By means of slopes, show that the following points lie on the same line $(a, 2b); (c, a+b); (2c-a, 2a)$

20. Find "k" so that the line joining $A(7,3); B(k,-6)$ and the line joining $C(-4,5); D(-6,4)$ are
- **a)** Parallel
- **b)** Perpendicular

21. Using slopes, show that the triangle with its vertices $A(6,1); B(2,7)$ & $C(-6,-7)$ is a right triangle

22. Find an equation of
- **a)** The horizontal line $(7,-9)$
- **b)** The vertical line through $(-5,3)$
- **c)** The line bisecting the 1st &3rd quadrants
- **d)** The line bisecting the 2nd & 4th quadrant

23. Find an equation of the line
- **a)** Through $A(-6,5)$ having slope 7
- **b)** Through $A(8,-3)$ having slope 0
- **c)** Through $(-6,5)$ having slope undefined
- **d)** Through $(-5,-3)$ & $(9,-1)$
- **e)** y – intercept: -7 and slope -5
- **f)** x – intercept: -3 and y – intercepts:4
- **g)** y – intercept: -9 and slope -4

24. Find an equation of the perpendicular bisector of the segment joining the points $A(3,5)$ & $B(9,8)$

25. Find an equation of the line through $(-4,-6)$ and perpendicular to a line having slope $\frac{-3}{2}$

26. Find an equation of the line through $(11,-5)$ and parallel to a line with slope -24

27. Convert each of the equation into slope intercept form & two intercept form
- **a)** $2x - 4y + 11 = 0$
- **b)** $4x + 7y - 2 = 0$
- **c)** $15y - 8x + 3 = 0$

28. In each of the following check whether the two lines are
- **a)** Parallel
- **b)** Perpendicular
- **c)** Neither || nor ⊥ar

- **i)** $2x + y - 3 = 0$ & $4x + 2y + 5 = 0$
- **ii)** $12x + 35y - 7 = 0$ & $105x - 36y + 11 = 0$
- **iii)** $4y + 2x - 1 = 0$ & $x - 2y - 7 = 0$

29. Find an equation of the line through $(-4,7)$ and parallel to the line $2x - 7y + 4 = 0$

30. Find an equation of the line through $(5,-8)$ and perpendicular to the join of $A(-15,-8); B(10,7)$

31. Find whether the given point lies above or below the given line
- **a)** $(5,8); 2x - 3y + 6 = 0$
- **b)** $(-7,6); 4x + 3y - 9 = 0$

32. Check whether the given points are on the same or opposite sides of the given line
- **a)** $(0,0)$ & $(5,8); 6x - y + 70 = 0$
- **b)** $(2,3)$ & $(-2,3); 3x - 5y + 8 = 0$

33. Find the distance from the point $P(6,-1)$ to the line $6x - 4y + 9 = 0$

34. Find the area of the triangular region whose vertices are $A(5,3), B(-2,2), C(4,2)$

35. Find the angle from the line with slope $-\frac{7}{3}$ to the line with slope $\frac{5}{2}$

36. Find the point of intersection of the line
- **a)** $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- **b)** $x + 4y - 12 = 0$ and $x - 3y + 3 = 0$

37. Find the condition that the line $y = m_1x + c_1; y = m_2x + c_2; y = m_3x + c_3$ are concurrent

38. Determine the value of "p" such that lines $2x - 3y - 1 = 0; 3x - y - 5 = 0$ and $3x + py + 8y = 0$ meet at a point

39. Check whether the lines $4x - 3y - 8 = 0; 3x - 4y - 6 = 0; x - y - 2 = 0$ are concurrent

40. Find measure of the angle between the lines represented by $x^2 - xy - 6y^2 = 0$

41. Find the lines also fined measure of the angle between them

- **a)** $10x^2 - 23xy - 5y^2 = 0$
- **b)** $3x^2 - 7xy + 2y^2 = 0$
- **c)** $9x^2 + 24xy + 16y^2 = 0$
- **d)** $2x^2 + 3xy - 5y^2 = 0$
- **e)** $6x^2 - 19xy + 15y^2 = 0$
- **f)** $x^2 + 2xy\sec\alpha + y^2 = 0$

Long Question

1. The mid – point of the sides of a triangle are $(1,-1); (-4,-3)$ & $(-1,1)$. Find coordinates of the vertices of the triangle

2. Find the point which equidistant from the points $A(5,3); B(-2,2)$ & $C(4,2)$. What is the radius of the circumcircle of the triangle ABC?

3. The points $(4,-2); (-2,4)$ & $(5,5)$ are the vertices of a triangle. Find in – centre of the triangle

4. The points $A(-1,2), B(3,-1)$ & $C(6,3)$ are consecutive vertices of a rhombus. Find the fourth vertex and show that the diagonals of the rhombus are perpendicular to each other

5. Find an equation of the sides, altitude and median of the triangle whose vertices are $A(-3,2), B(5,4)$ & $C(3,-8)$.

6. The points $A(-1,2), B(6,3)$ & $C(2,-4)$are vertices of a triangle. Show that the line joining the mid – point D of AB and the mid – point E of AC is parallel to BC & $DE = \frac{1}{2}BC$

7. Find the distance between the given parallel lines. Sketch the lines. Also find an equation of the parallel line lying midway between them

- **a)** $3x - 4y + 3 = 0; 3x - 4y + 7 = 0$
- **b)** $12x + 5y - 6 = 0; 12x + 5y + 13 = 0$

8. Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x – and y – intercepts of each is 3

9. One vertex of a parallelogram is $(1,4)$; the diagonals intersect at $(2,1)$ and the sides having slope 1 and $-\frac{1}{7}$. Find the other three vertices

10. Find an equation of the line through

- **a)** The point $(2,-9)$ & the intersection of the line $2x + 5y - 8 = 0; 3x - 4y - 6 = 0$
- **b)** The intersection of the lines $x - y - 4 = 0; 7x + y + 20 = 0$ and parallel & perpendicular to the line $6x + y - 14 = 0$
- **c)** Through the intersection of the lines $x + 2y + 3 = 0; 3x + 4y + 7 = 0$ and making equal intercepts on the axes

11. Find an equation of the line through the intersection of $16x - 10y - 33 = 0; 12x + 14y + 29 = 0$ and the intersection of $x - y + 4 = 0; x - 7y + 2 = 0$

12. Find the coordinates of the vertices of the triangle formed by the lines $x - 2y - 6 = 0; 3x - y + 3 = 0; 2x + y - 4 = 0$ also find measure of the angles of the triangle.

13. Find the angle measured from the l_1 to the line l_2 also find acute angle where

- **a)** l_1: Joining $(2,7)$ & $(7,10)$ l_2: Joining $(1,1)$ & $(-5,3)$
- **b)** l_1: Joining $(-9,-1)$ & $(3,-5)$ l_2: Joining $(2,7)$ & $(-6,-7)$

14. Find the interior angles of the triangle whose vertices are

- **a)** $A(-2,11), B(-6,-3), C(4,-9)$
- **b)** $A(2,-5), B(-4,-3), C(-2,5)$

15. Find the area of the region bounded by the triangle whose sides are $7x - y - 10 = 0; 10x + y - 41 = 0; 3x + 2y + 3 = 0$

16. Find a join equation of the line through the origin and perpendicular to the lines $x^2 - 2xy\tan\alpha - y^2 = 0$

17. Find a join equation of the lines through the origin and perpendicular to the lines $ax^2 - 2h\,xy + by^2 = 0$

18. Find the area of the region bounded by $10x^2 - xy - 21y^2 = 0$ and $x + y + 1 = 0$

Chapter No.05

Linear Inequalities Linear Programming

• Short Questions

1. Defined the following

- **a)** Linear inequality
- **b)** Problem Constraint
- **c)** Non – negative constraint or decision variable
- **d)** Feasible region
- **e)** Feasible solution
- **f)** Feasible solution set
- **g)** Convex region
- **h)** Linear programming
- **i)** Objective function
- **j)** Optimal solution
- **k)** State the theorem of linear programming
- **l)** Corner point or vertex of the solution region
- **m)** Associative equation of an inequality

2. Graph the solution set of each of the following linear inequality in xy − plane

- **a)** $2x + y \leq 6$
- **b)** $3x + 7y \geq 21$
- **c)** $3x - 2y \geq 6$
- **d)** $5x - 4y \leq 20$
- **e)** $2x + 1 \geq 0$
- $3y - 4 \leq 0$

3. Indicated the solution set of the following system of linear inequalities by shading

- **a)** $2x - 3y \leq 6$; $2x + 3y \geq 12$
- **b)** $x + y \geq 5$; $x - y \leq 1$
- **c)** $3x + 7y \geq 21$; $x - y \leq 2$
- **d)** $4x - 3y \leq 12$; $x \geq -\frac{3}{2}$
- **e)** $3x + 7y \geq 21$; $y \leq 4$

4. Shaded the feasible region of $5x + 7y \leq 35$

5. Is (3,2) in the solution of the inequality $x - y \geq 1$?

6. Find the associative equation for the linear inequality $3x + 7y \geq 21$

Long Questions

1. Graph the feasible region of the following system of linear inequalities and find the corner points in each case

- **a)** $2x - 3y \leq 6$; $2x + 3y \leq 12$; $x \geq 0; y \geq 0$
- **b)** $x + y \leq 5$; $-2x + y \leq 2$; $x \geq 0; y \geq 0$
- **c)** $x + y \leq 5$; $-2x + y \geq 2$; $x \geq 0$
- **d)** $3x + 7y \leq 21$; $x - y \leq 3$; $x \geq 0; y \geq 0$
- **e)** $3x + 2y \geq 6$; $x + y \leq 4$; $x \geq 0; y \geq 0$
- **d)** $5x + 7y \leq 35$; $x - 2y \leq 4$; $x \geq 0; y \geq 0$

2. Find the minimum and maximum values of f & φ defined as:
$f(x, y) = 4x + 5y$; $\varphi(x, y) = 4x + 6y$ under the constraint
$2x - 3y \leq 6; 2x + y \geq 2; 2x + 3y \leq 12; x \geq 0, y \geq 0$

3. Maximize $f(x, y) = 2x + 5y$ subject to the constraints $2y - x \leq 8; x - y \leq 4; x \geq 0; y \geq 0$

4. Maximize $f(x, y) = x + 3y$ subject to the constraints $2x + 5y \leq 30; 5x + 4y \leq 20; x \geq 0; y \geq 0$

5. Maximize $z = 2x + 3y$ subject to the constraints $3x + 4y \leq 12; 2x + y \leq 4; 4x - y \leq x \geq 0; y \geq 0$

6. Minimize $z = 2x + y$ subject to the constraints $x + y \geq 3; 7x + 5y \leq 35; x \geq 0; y \geq 0$

7. Maximize $f(x, y) = 2x + 3y$ subject to the constraints $2x + y \leq 8; x + 2y \leq 14; x \geq 0; y \geq 0$

8. Minimize $z = 3x + y$ subject to the constraints $3x + 5y \geq 15; x + 6y \geq 9; x \geq 0; y \geq 0$

Chapter No.06

Conic Section

• Short Questions

Exercise 6.1

1. Find the equation of circle with
   (a) Centre at $(-3,5)$ and radius 7 (Example) (b) Centre at $(5,-2)$ and radius 4
   (c) Centre at $(\sqrt{2},-3\sqrt{3})$ and radius $2\sqrt{2}$ (d) End of a diameter at $(-3,2)$ & $(5,-6)$
2. Find the centre and radius of the circle with given equation
   (a) $x^2+y^2+12x-10y=0$ (b) $5x^2+5y^2+14x+12y-10=0$
   (c) $x^2+y^2-6x+4y+13=0$ (d) $4x^2+4y^2-8x+12y-25=0$
3. Show that the equation $5x^2+5y^2+24x+36y+10=0$ represents a circle. Also find its centre and radius.
4. Find equation of circle of radius 'a' and lying in the second quadrant such that it is tangent to both the axis

Exercise 6.2

5. Write down equations of the tangent and normal to the circle
   (a) $x^2+y^2=25$ at $(4,3)$ (b) $3x^2+3y^2+5x-13y+2=0$ at $\left(1,\frac{10}{3}\right)$
6. Check the position of the point $(5,6)$ w.r.t. to the circle
   (a) $x^2+y^2=81$ (b) $2x^2+2y^2+12x-8y+1=0$
7. Find the length of the tangent drawn from the point
   (a) $(-5,4)$ to the circle $5x^2+5y^2-10x+15y-131=0$
   (b) $(-5,10)$ to the circle $5x^2+5y^2+14x+12y-10=0$
8. Determine whether the point $P(-5,6)$ lies outside, on or inside the circle $x^2+y^2+4x-6y-12=0$

Exercise 6.4

9. Find the focus, vertex and directrix of the parabola. Sketch its graph
   (a) $y^2=8x$ (b) $x^2=-16y$ (c) $x^2=5y$
   (d) $y^2=-12x$ (e) $x^2=4(y-1)$ (f) $y^2=-8(x-3)$
   (g) $(x-1)^2=8(y+2)$ (h) $y=6x^2-1$ (i) $x+8-y^2+2y=0$
   (j) $x^2-4x-8y+4=0$
10. Write an equation of the parabola with given elements
    (a) Focus $F(-3,1)$; DTX $x = 3$ (b) Focus $F(2,5)$; DTX $y = 1$
    (c) Focus $F(-1,0)$; Vertex $V(-1,2)$ (d) Axis $y = 0$, through (2, 1) (11,– 2)
11. Find an equation of the parabola having its focus at the origin and directrix parallel to
    (a) The x – axis (b) The y – axis
12. Show that the ordinate at any point P of the parabola is a mean proportional between the length of the latus – rectuim and the abscissa of P
13. A parabolic arch has a 100m base and height 25 m. Find the height of the arch at the point 30 m from the centre of the base

Exercise 6.5

14. Find an equation of the ellipse with given data
    (a) Foci $(\pm 3,0)$ and minor axis of length 10
    (b) Foci $(0,-1)$ and $(0,-5)$ and major axis of length 6
    (c) Foci $(\pm 3\sqrt{3},0)$ and Vertices $(\pm 6,0)$
    (d) Vertices $(-1,1),(5,1)$ and Foci $(4,1),(1,1)$
    (e) Foci $(\pm\sqrt{5},0)$ and passing through the point $\left(\frac{3}{2},\sqrt{3}\right)$
    (f) Vertices $(0,\pm 5)$; eccentricity $\frac{3}{5}$

(g) Centre (0, 0); Focus (0, – 3) and Vertex (0, 4)

Exercise 6.6

15. Find an equation of the hyperbola with the given data
(a) Centre (0, 0), Focus (6, 0) and Vertex (4, 0)
(b) Foci $(\pm 5,0)$, Vertex (3, 0)
(c) Foci $\left(2 \pm 5\sqrt{2}, -7\right)$, length of transverse axis 10
(d) Foci $(0, \pm 6), e = 2$

Exercise 6.7

16. Find equation of the tangent and normal to each of the following indicated point
(a) $y^2 = 4ax$ at $(at^2, 2at)$
(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$

Long Questions

1. Write an equation of the circle that passes through given points
(a) $A(4,5), B(-4,-3)$ & $C(8,-3)$
(b) $A(-7,7), B(5,-1), C(10,0)$
(c) $A(a,0), B(0,b), C(0,0)$
(d) $A(5,6), B(-3,2), C(3,-4)$

2. Find equation of circle passing through
(a) $A(3,-1), B(0,1)$ and having centre at $4x - 3y - 3 = 0$
(b) $A(-3,1)$ with radius 2 and centre at $2x - 3y + 3 = 0$
(c) $A(5,1)$ and tangent to line 2x – y – 10 = 0 at $B(3,-4)$
(d) $A(1,4), B(-1,8)$ and tangent to line $x + 3y - 3 = 0$

3. Show that the line 3x – 2y = 0 and 2x + 3y – 13 = 0 are tangent to the circle $x^2 + y^2 + 6x - 14y = 0$

4. Show that the circle $x^2 + y^2 + 2x - 2y - 7 = 0$ & $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally

5. Show that the circle $x^2 + y^2 + 2x - 8 = 0$ and $x^2 + y^2 - 6x + 6y - 46 = 0$ touch internally

6. Find equation of the circle of radius 2 and tangent to line x – y – 4 = 0 at A(1,– 3)

7. Find the co – ordinates of the points of intersection of line 2x + y = 5 and the circle $x^2 + y^2 + 2x - 9 = 0$. Also find the length of the intercepted chord

8. Write equation of two tangents from (2, 3) to the circle $x^2 + y^2 = 9$

9. Let $P(x_1, y_1)$ be a point outside of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then length of tangent from P to circle is given by $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

10. Find the co – ordinates of the points intersection of the line $x + 2y = 6$ with the circle $x^2 + y^2 - 2x - 2y - 39 = 0$

11. Find the equation of tangents to the circle $x^2 + y^2 = 2$
(a) Parallel to the line $x - 2y + 1 = 0$
(b) Perpendicular to the line $3x + 2y = 6$

12. Find the equation of tangents drawn from
(a) $(-1,2)$ to $x^2 + y^2 + 4x + 2y = 0$
(b) $(-7,-2)$ to $(x+1)^2 + (y-2)^2 = 26$
Also find the points of contact.

13. Find the equation of the chord of contact of the tangents drawn from (4, 5) to the circle $2x^2 + 2y^2 - 8x + 12 + 21 = 0$

14. Prove that the straight line drawn from the centre of a circle perpendicular to a tangent passes through the point of tangency

15. Show that an equation of the parabola with focus at $(a\cos\alpha, a\sin\alpha)$ and DTX $x\cos\alpha + y\sin\alpha + a = 0$ is $(x\sin\alpha - y\cos\alpha)^2 = 4a(x\cos\alpha + y\sin\alpha)$

16. Find the Centre, Foci, Eccentricity, Vertices and DTX of the ellipse whose equation is given
(a) $x^2 + 4y^2 = 16$
(b) $9x^2 + y^2 = 18$
(c) $25x^2 + 9y^2 = 225$
(d) $\frac{(2x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

17. Let "a" be a positive number and $0 < c < a$, Let $F(-c,0), F'(c,0)$ be two given points. Prove that the locus of point P(x, y) such that $|PF| + |PF'| = 2a$ is an ellipse

18. Prove that the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$

19. Find the Centre, Foci, Eccentricity, Vertices and equations of directrices of the following

(a) $x^2-y^2=9$
(b) $\frac{x^2}{4}-\frac{y^2}{9}=1$
(c) $\frac{y^2}{16}-\frac{x^2}{9}=1$
(d) $\frac{y^2}{4}-x^2=1$
(e) $\frac{(x-1)^2}{2}-\frac{(y-1)^2}{9}=1$
(f) $9x^2-12x-y^2-2y+2=0$
(g) $9x^2-y^2-36x-6y+18=0$

20. Discuss and sketch the graph of the equation $25x^2-16y^2=400$

21. Find the eccentricity, the coordinates of the vertices and foci of the asymptotes of the hyperbola $\frac{y^2}{16}-\frac{x^2}{49}=1$ Also sketch its graph

22. Discuss and sketch the graph of the equation $4x^2-8x-y^2-2y-1=0$

23. Let $0<a<c$, Let $F(c,0), F'(-c,0)$ be two fixed points. Show that the set of points P(x, y) such that $|PF|-|PF'|=\pm 2a$ is hyperbola $\frac{x^2}{a^2}-\frac{y^2}{c^2-a^2}=1$

24. For any point on a hyperbola the difference of its distance from the points (2, 2) and (10, 2) is 6. Find an equation of hyperbola

25. Write equation of the tangent and normal to the parabola $x^2=16y$ at the point whose abscissa is 8

26. Write equations of the tangent and normal to the conic $\frac{x^2}{8}+\frac{y^2}{9}=1$ at the point $\left(\frac{8}{3},1\right)$

27. Find the equations of tangents to the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ which are parallel to line $2x-4y+5=0$

28. Find the points of intersection of the given conics

(a) $\frac{x^2}{18}+\frac{y^2}{8}=1$ & $\frac{x^2}{3}-\frac{y^2}{3}=1$
(b) $x^2+y^2=8$ & $x^2-y^2=1$
(c) $3x^2-4y^2=12$ & $3y^2-2x^2=7$
(d) $3x^2+5y^2=60$ and $9x^2+y^2=124$

29. Find an equation of the tangent to the conic $x^2-xy+y^2-2=0$ at the point whose ordinate is $\sqrt{2}$

30. Show that

(a) $10xy+8x-15y-12=0$ and
(b) $6x^2+xy-y^2-21x-8y+9=0$

Each represents a pair of straight lines and find an equation of each line.

Chapter No.07

Vectors

• Short Questions

1. Define the following

a) Vector & Scalar quantity
b) Multiplication of vector by a scalar
c) Equal & Parallel vector
d) Position vector
e) Direction angles & direction cosine
f) Scalar product
g) Vector product
h) Two property of scalar product
i) Two property of vector product
j) Scalar triple product of vector

2. Prove that $\underline{r}=\frac{qa+pb}{p+q}$

3. Write the vector $\overrightarrow{PQ}$ in the form of $x\underline{i}+y\underline{j}$

a) $P(2,3), Q(6,-2)$
b) $P(0,5), Q(-1,-6)$

4. Find the magnitude of the vector $\underline{u}$

a) $\underline{u}=2\underline{i}-7\underline{j}$
b) $\underline{u}=\underline{i}+\underline{j}$
c) $\underline{u}=[3,-4]$

5. Find the sum of the vectors $\overrightarrow{AB}$ & $\overrightarrow{CD}$, given the four points $A(1,-1), B(2,0), C(-1,2)$ & $D(-2,2)$.
6. Find the vector from the point A to the origin where $\overrightarrow{AB} = 4\underline{i} - 2\underline{j}$ and B point $(-2,5)$
7. Find the unit vector in the direction of the vector given below
   a) $\underline{v} = 2\underline{i} - \underline{j}$ b) $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$ c) $\underline{v} = \frac{-\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$
8. If O is the origin and $\overrightarrow{OP} = \overrightarrow{AB}$, find the point P when A and B are $(-3,7)$ & $(1,0)$ respectively
9. Use vectors, to show that ABCD is a parallelogram, when the points A, B, C & D are respectively $(0,0), (a,0), (b,c)$ & $(b-a,c)$.
10. If $\overrightarrow{AB} = \overrightarrow{CD}$. Find the coordinate of the points A, B, C & D are $(1,2), (-2,5), (4,11)$ respectively
11. Find the position vector of the
    a) Point C with position vector $2\underline{i} - 3\underline{j}$ and point D with position vector $3\underline{i} + 2\underline{j}$ in the ratio $4:3$
    b) Point E with position vector $5\underline{j}$ and point F with position vector $4\underline{i} + \underline{j}$ in the ratio $2:5$
12. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
13. Let $A(2,5); B(-1,1)$ & $C(2,-6)$. Find
    a) $2\overrightarrow{AB} - \overrightarrow{CB}$ b) $2\overrightarrow{CB} - 2\overrightarrow{CA}$
14. Let $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}; \underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}; \underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$. Find the indicated vector or number
    a) $\underline{u} + 2\underline{v} + \underline{w}$ b) $|3\underline{v} + \underline{w}|$
15. Find the magnitude of the vector $\underline{v}$ and write the direction cosines of $\underline{v}$
    a) $\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$ b) $\underline{v} = 4\underline{i} - 5\underline{j}$
16. Find "α" so that $\left|\alpha\underline{i} + (\alpha+1)\underline{j} + 2\underline{k}\right| = 3$
17. Find a vector whose
    a) Magnitude is 4 and is parallel to the $2\underline{i} - 3\underline{j} + 6\underline{k}$
    b) Magnitude is 2 and is parallel to the $-\underline{i} + \underline{j} + \underline{k}$
18. If $\underline{u} = 2\underline{i} + 3\underline{j} + 4\underline{k}; \underline{v} = -\underline{i} + 3\underline{j} - \underline{k}$ & $\underline{w} = \underline{i} + 6\underline{j} + z\underline{k}$ represent the sides of a triangle. Find the value of "z"
19. The position vectors of the points A, B, C and D $2\underline{i} - \underline{j} + \underline{k}; 3\underline{i} + \underline{j}; 2\underline{i} + 4\underline{j} - 2\underline{k}$ & $-\underline{i} - 2\underline{j} + \underline{k}$ respectively. Show that $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.
20. If $\underline{u}$ & $\underline{v}$ are parallel then
    a) Find two vectors of the length 2 parallel vector $\underline{v} = 2\underline{i} - 4\underline{j} + 4\underline{k}$
    b) Find the constant α so that the vector $\underline{v} = \underline{i} - 3\underline{j} + 4\underline{k}$ & $\underline{w} = \alpha\underline{i} + 9\underline{j} - 12\underline{k}$ are parallel
    c) Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$
    d) Find "a" and "b" so that the vectors $3\underline{i} - \underline{j} + 4\underline{k}$ and $a\underline{i} + b\underline{j} - 2\underline{k}$ are parallel
21. Find the direction cosines for the given vector
    a) $\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$ b) $\underline{v} = 6\underline{i} - 2\underline{j} + \underline{k}$ c) $\overrightarrow{PQ}$ where $P(2,1,5); Q(1,3,1)$
22. Which of the following triples can be in the direction angle of a single vector
    a) $45^0, 45^0, 60^0$ b) $30^0, 45^0, 60^0$ c) $45^0, 60^0, 60^0$
23. Find the cosine of the angle θ between $\underline{u}$ & $\underline{v}$
    a) $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}; \underline{v} = 2\underline{i} - \underline{j} + \underline{k}$ b) $\underline{u} = [2,-3,1], \underline{v} = [2,4,1]$
24. Calculate the projection of $\underline{a}$ along $\underline{b}$ and projection of $\underline{b}$ along $\underline{a}$ when
    a) $\underline{a} = \underline{i} - \underline{k}; \underline{b} = \underline{j} + \underline{k}$ b) $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}; \underline{b} = -2\underline{i} - \underline{j} + \underline{k}$
25. Find a real number "α" so that the vector $\underline{u}$ and $\underline{v}$ are perpendicular
    a) $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}; \underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$ b) $\underline{u} = \alpha\underline{i} + 2\alpha\underline{j} - \underline{k}; \underline{v} = \underline{i} + \alpha\underline{j} + \underline{k}$

26. Find the number "z" so that the triangle with vertices $A(1,-1,0); B(-2,2,1)$ & $C(0,2,z)$ is a right triangle with right angle at C

27. If $\underline{v}$ is a vector for which $\underline{v}.\underline{i} = 0$; $\underline{v}.\underline{j} = 0$; $\underline{v}.\underline{k} = 0$, find $\underline{v}$.

28. Show that vectors

- **a)** $3\underline{i} - 2\underline{j} + \underline{k}$; $\underline{i} - 3\underline{j} + 5\underline{k}$ & $2\underline{i} + \underline{j} - 4\underline{k}$ form a right angle
- **b)** The points $P(1,3,2), Q(4,1,4)$ & $R(6,5,5)$ form a right triangle

29. Find the $\underline{a} \times \underline{b}$ & $\underline{b} \times \underline{a}$ if

- **a)** $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$; $\underline{b} = \underline{i} - \underline{j} + \underline{k}$
- **b)** $\underline{a} = -4\underline{i} + \underline{j} - 2\underline{k}$; $\underline{b} = 2\underline{i} + \underline{j} + \underline{k}$

30. Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$. Also find sine of the angle between them

- **a)** $\underline{a} = 2\underline{i} - 6\underline{j} + 3\underline{k}$; $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$
- **b)** $\underline{a} = \underline{i} + \underline{j}$; $\underline{b} = \underline{i} - \underline{j}$

31. Find the area of the triangle determined by the point P, Q, and R

- **a)** $P(0,0,0)$; $Q(2,3,2)$; $R(-1,1,4)$
- **b)** $P(1,-1,-1); Q(2,0,1); R(0,2,1)$

32. Find the area of parallelogram, whose vertices are

- **a)** $A(0,0,0); B(1,2,3); C(2,-1,1); D(3,1,4)$
- **b)** $A(-1,11); B(-1,2,2); C(-3,4,-5); D(-3,5,5,4)$

33. Which vectors, if any are perpendicular or parallel

- **a)** $\underline{u} = 5\underline{i} - \underline{j} + \underline{k}$; $\underline{v} = \underline{j} - 5\underline{k}$; $\underline{w} = -15\underline{i} + 3\underline{j} - 3\underline{k}$
- **b)** $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$; $\underline{v} = -\underline{i} + \underline{j} + \underline{k}$; $\underline{w} = -\frac{\pi}{2}\underline{i} - \pi\underline{j} - \frac{\pi}{2}\underline{k}$

34. Prove that $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$

35. If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$

36. If $\underline{a} \times \underline{b} = 0$ and $\underline{a}.\underline{b} = 0$, what conclusion can be drawn about $\underline{a}$ or $\underline{b}$?

37. Prove that four points $A(-3,5,-4); B(-1,1,1); C(-1,2,2)$ & $D(-3,4,-5)$ are coplanar.

38. Prove that the points whose position vector
$A(-6\underline{i} + 3\underline{j} + 2\underline{k}), B(3\underline{i} - 2\underline{j} + 4\underline{k}), C(5\underline{i} + 7\underline{j} + +3\underline{k}), D(-13\underline{i} + 17\underline{j} - \underline{k})$ are coplanar

39. Find the volume of the parallelepipied for which the given vectors are three edges

- **a)** $\underline{u} = 3\underline{i} + 2\underline{k}$; $\underline{v} = \underline{i} + 2\underline{j} + \underline{k}$; $\underline{w} = -\underline{j} + 4\underline{k}$
- **b)** $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$; $\underline{v} = 2\underline{i} - \underline{j} - \underline{k}$; $\underline{w} = \underline{j} + 1\underline{k}$

40. Prove that the vectors $\underline{i} - 2\underline{j} + 3\underline{k}; -2\underline{i} + 3\underline{j} - 4\underline{k}$ & $\underline{i} - 3\underline{j} + 5\underline{k}$ are coplanar

41. Find the constant α such that the vectors are coplanar

- **a)** $\underline{i} - \underline{j} + \underline{k}$; $\underline{i} - 2\underline{j} - 3\underline{k}$ & $3\underline{i} - \alpha\underline{j} + 5\underline{k}$
- **b)** $\underline{i} - 2\alpha\underline{j} - \underline{k}$; $\underline{i} - \underline{j} + 2\underline{k}$ & $\alpha\underline{i} - 2\underline{j} + \underline{k}$

42. Find the value of

- **a)** $2\underline{i} \times 2\underline{j}.\underline{k}$
- **b)** $3\underline{j}.\underline{k} \times \underline{i}$
- **c)** $[\underline{k}\ \underline{i}\ \underline{j}]$
- **d)** $[\underline{i}\ \underline{i}\ \underline{k}]$

43. Prove that $\underline{u}.(\underline{v} \times \underline{w}) + \underline{v}.(\underline{w} \times \underline{u}) + \underline{w}.(\underline{u} \times \underline{v}) = 3\underline{u}.(\underline{v} \times \underline{w})$

44. Find the work done by a constant force $\underline{F} = 2\underline{i} + 4\underline{j}$, if its points of application to a body moves it from $A(1,1)$ to $B(4,6)$

45. Find the work done, if the point at which the constant force $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object, moves from $P_1(3,1,-2)$ to $P_2(2,4,6)$

46. A particle acted by constant forces $4\underline{i} + \underline{j} - 3\underline{k}$ and $3\underline{i} - \underline{j} - \underline{k}$ is displaced from $A(1,2,3)$ to $B(5,4,1)$. Find the work done

47. A force of magnitude 6 unit acting parallel to $2\underline{i} - 2\underline{j} + \underline{k}$ displace the point of application from $(1,2,3)$ to $(5,3,7)$. Find the work done

48. A force $\underline{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ is applied at the point $(1,-1,2)$. Find the moment of the force about the point $(2,-1,3)$

49. A force $\underline{F} = 4\underline{i} - 3\underline{k}$, passes through the point $A(2,-2,5)$. Find the moment of $\underline{F}$ about the point $B(1,-3,1)$.

50. A force $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$, is applied at $P(1,-2,3)$. Find the moment about the point $Q(2,1,1)$.

Long Questions

1. Prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side and half as long

2. Prove that the line segments joining the mid points of the sides of a quadrilateral taken in order form a parallelogram

3. Use vectors, to prove that the diagonals of a parallelogram bisect each other

4. Prove that in any triangle ABC

- **a)** $a^2 = b^2 + c^2 - 2bc\cos A$
- **b)** $a = b\cos C + c\cos B$
- **c)** $b = c\cos A + a\cos C$
- **d)** $c = a\cos B + b\cos A$
- **e)** $b^2 = c^2 + a^2 - 2ac\cos B$
- **f)** $c^2 = a^2 + b^2 - 2ab\cos C$

5. Show that mid-point of hypotenuse a right triangle is equidistant from its vertices

6. Prove that perpendicular bisectors of the sides of a triangle are concurrent

7. Prove that the altitudes of a triangle are concurrent

8. Prove that the angle in a semi-circle is a right angle

9. Prove that

- **a)** $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- **b)** $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- **c)** $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- **d)** $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

10. In any triangle ABC, prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

11. Find the volume of the tetrahedron with the vertices

- **a)** $(0,1,2); (3,2,1); (1,2,1)$ & $(5,5,6)$
- **b)** $(2,1,8); (3,2,9); (2,1,4)$ & $(3,3,10)$

12. A particle is displaced from the point $A(5,-5,-7)$ to the point $B(6,2,-2)$ under the action of constant forces defined by $10\underline{i} - \underline{j} + 11\underline{k}$; $4\underline{i} + \underline{j} + 9\underline{k}$ and $-2\underline{i} + \underline{j} - 9\underline{k}$. Show that the total work done by the forces is 102 units