Ch #01

Quadratic Equation

Ex #1.1

Que #01

Write the following quadratic Equations in standard form and point out pure quadratic Equations.

i $(x+7)(x-3) = -7$

$(x+7)(x-3) = -7$

$x(x-3) + 7(x-3) = -7$

$x^2 - 3x + 7x - 21 = -7$

$x^2 + 4x - 21 + 7 = 0$

$x^2 + 4x - 14 = 0$

The above equation is quadratic Equation.

ii $\frac{x^2+4}{3} - \frac{x}{7} = 1$

$\frac{x^2+4}{3} - \frac{x}{7} = 1$

Multiply both sides by "21"

$21 \times \frac{x^2+4}{3} - \frac{x}{7} \times 21 = 1 \times 21$

$7(x^2+4) - 3x = 21$

$7x^2 + 28 - 3x = 21$

$7x^2 - 3x + 28 - 21 = 0$

$7x^2 - 3x + 7 = 0$

The above equation is quadratic Equation.

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

iii $\frac{x}{x+1} + \frac{x+1}{x} = 6$

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$

$$\frac{x^2 + (x^2+1)^2}{x(x+1)} = 6$$

$$x^2 + x^2 + 2x + 1 = 6x(x+1)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$2x^2 - 6x^2 + 2x - 6x + 1 = 0$$

$$-4x^2 - 4x + 1 = 0$$

$$-(4x^2 + 4x - 1) = 0$$

$$4x^2 + 4x - 1 = 0$$

The above Equation is quadratic Equation.

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

iv $\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$

$$\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$$

$$\frac{x+4}{x-2} - \frac{x-2}{x} = -4$$

$$\frac{x(x+4) - (x-2)^2}{x(x-2)} = -4$$

$$x^2 + 4x - (x^2 - 4x + 4) = -4(x^2 - 2x)$$

$$x^2 + 4x - x^2 + 4x - 4 = -4x^2 + 8x$$

$$8x - 4 + 4x^2 - 8x = 0$$

$$4(x^2 - 1) = 0$$

$$x^2 - 1 = 0$$

This Equation is pure quadratic Equation.

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

v $\quad \dfrac{x+3}{x+4} - \dfrac{x-5}{x} = 1$

$$\frac{x+3}{x+4} - \frac{x-5}{x} = 1$$

$$\frac{x(x+3) - (x+4)(x-5)}{x(x+4)} = 1$$

$$(x^2+3x) - x(x-5) - 4(x-5) = x(x+4)$$

$$x^2+3x-x^2+5x-4x+20 = x^2+4x$$

$$x^2-x^2+3x+5x-4x+20 = x^2+4x$$

$$4x+20-x^2-4x = 0$$

$$-x^2+20 = 0$$

$$-(x^2-20) = 0$$

$$x^2-20 = 0$$

The above Equation is a pure quadratic Equation.

vi $\quad \dfrac{x+1}{x+2} + \dfrac{x+2}{x+3} = \dfrac{25}{12}$

$$\frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}$$

$$\frac{(x+1)(x+3) + (x+2)^2}{(x+2)(x+3)} = \frac{25}{12}$$

$$\frac{x(x+3)+1(x+3)+(x^2+4x+4)}{(x+2)(x+3)} = \frac{25}{12}$$

$$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6} = \frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6} = \frac{25}{12}$$

$$25(x^2+5x+6) = 12(2x^2+8x+7)$$

$$25x^2+125x+150 = 24x^2+96x+84$$

$$25x^2-24x^2+125x-96x+150-84 = 0$$

$x^2 + 29x + 66 = 0$

The above Equation is quadratic Equation.

Q.No.2 Solve by Factorization:

i $x^2 - x - 20 = 0$

$x^2 - x - 20 = 0$

$x^2 - 5x + 4x - 20 = 0$

$x(x-5) + 4(x-5) = 0$

$(x-5)(x+4) = 0$

Either $x - 5 = 0$ or $x + 4 = 0$

$x = 5$ $x = -4$

Thus, solution set $= \{5, -4\}$

ii $3y^2 = y(y-5)$

$3y^2 = y(y-5)$

$3y^2 = y^2 - 5y$

$3y^2 - y^2 + 5y = 0$

$2y^2 + 5y = 0$

$y(2y+5) = 0$

Either $y = 0$ or $2y + 5 = 0$

$2y = -5$

$y = -5/2$

Thus solution set $= \{0, -5/2\}$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

iii $4-32x = 17x^2$

$4-32x = 17x^2$

$17x^2+32x-4=0$

$17x^2+34x-2x-4=0$

$17x(x+2)-2(x+2)=0$

$(x+2)(17x-2)=0$

Either $x+2=0$ or $17x-2=0$

$x=-2$ $\quad 17x=2$

$x=\frac{2}{17}$

Thus solution set $=\{\frac{2}{17}, -2\}$

iv $x^2-11x=152$

$x^2-11x = 152$

$x^2-11x-152=0$

$x^2-19x+8x-152=0$

$x(x-19)+8(x-19)=0$

$(x+8)(x-19)=0$

Either $x+8=0$ or $x-19=0$

$x=-8$ $\quad x=19$

Thus solution set $=\{-8, 19\}$

v $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

$\frac{(x+1)}{x}+\frac{x}{x+1}=\frac{25}{12}$

$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$

$\frac{x^2+2x+1+x^2}{x^2+x}=\frac{25}{12}$

$25(x^2+x)=12(2x^2+2x+1)$

$25x^2+25x=24x^2+24x+12$

$$25x^2 - 24x^2 + 25x - 24x - 12 = 0$$

$$x^2 + x - 12 = 0$$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

$$x^2 + 4x - 3x - 12 = 0$$

$$x(x+4) - 3(x+4) = 0$$

$$(x-3)(x+4) = 0$$

Either $x - 3 = 0$ OR $x + 4 = 0$

$x = 3$ $\qquad$ $x = -4$

Thus solution set $= \{3, -4\}$

vi $\quad \dfrac{2}{x-9} = \dfrac{1}{x-3} - \dfrac{1}{x-4}$

$$\frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$$

$$\frac{2}{x-9} = \frac{(x-4) - (x-3)}{(x-3)(x-4)}$$

$$\frac{2}{x-9} = \frac{x-4-x+3}{x^2-4x-3x+12}$$

$$\frac{2}{x-9} = \frac{-1}{x^2-7x+12}$$

$$2(x^2-7x+12) = -1(x-9)$$

$$2x^2 - 14x + 24 = -x + 9$$

$$2x^2 - 14x + 24 + x - 9 = 0$$

$$2x^2 - 13x + 15 = 0$$

$$2x^2 - 10x - 3x + 15 = 0$$

$$2x(x-5) - 3(x-5) = 0$$

$$(2x-3)(x-5) = 0$$

Either $2x - 3 = 0$ or $x - 5 = 0$

$2x = 3$ $\qquad$ $x = 5$

$x = \frac{3}{2}$

Thus solution set $= \{5, 3/2\}$

Q.No.03

Solve the following equations by completing Square:

i $7x^2+2x-1=0$

$$7x^2+2x-1=0$$

$$7x^2+2x=1$$

$$\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7}$$

$$x^2+\frac{2x}{7}=\frac{1}{7}$$

$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{8}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=-\frac{1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1\pm2\sqrt{2}}{7}$$

Thus, solution set $=\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

ii $ax^2+4x-a=0$

$ax^2+4x-a=0$

$ax^2+4x=a$

$\frac{ax^2}{a}+\frac{4x}{a}=\frac{a}{a}$

$x^2+\frac{4x}{a}=1$

$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$

$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$

$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$

Taking square root on both sides,

$x+\frac{2}{a}=\pm\sqrt{\frac{a^2+4}{a^2}}$

$x=-\frac{2}{a}\pm\frac{\sqrt{a^2+4}}{a}$

$x=\frac{-2\pm\sqrt{a^2+4}}{a}$

Thus solution set $=\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

iii $11x^2-34x+3=0$

$11x^2-34x+3=0$

$11x^2-34x=-3$

$\frac{11x^2}{11}-\frac{34}{11}x=\frac{-3}{11}$

$x^2-\frac{34}{11}x=-\frac{3}{11}$

$(x)^2-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=\frac{-3}{11}+\left(\frac{34}{22}\right)^2$

$\left(x-\frac{34}{22}\right)^2=\frac{-3}{11}+\frac{1156}{484}$

$\left(x-\frac{34}{22}\right)^2=\frac{-132+1156}{484}$

$$\left(x-\frac{34}{22}\right)^2 = \frac{1024}{484}$$

Taking square root on both sides.

$$\sqrt{\left(x-\frac{34}{22}\right)^2} = \pm\sqrt{\frac{1024}{484}}$$

$$x-\frac{34}{22} = \pm\frac{32}{22}$$

$$x = +\frac{34}{22} \pm \frac{32}{22}$$

$$x = \frac{34 \pm 32}{22}$$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

$$x = \frac{34+32}{22} \quad , \quad x = \frac{34-32}{22}$$

$$x = \frac{66}{22} \quad , \quad x = \frac{2}{22}$$

$$x = 3 \quad , \quad x = \frac{1}{11}$$

Thus solution set $= \{3, 1/11\}$

iv $\quad \ell x^2 - mx + n = 0$

$$\ell x^2 - mx + n = 0$$

$$\ell x^2 - mx = -n$$

$$\frac{\ell x^2}{\ell} - \frac{mx}{\ell} = -\frac{n}{\ell}$$

$$x^2 - \frac{mx}{\ell} = -\frac{n}{\ell}$$

$$x^2 + 2(x)\left(\frac{m}{2\ell}\right) + \left(\frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \left(\frac{m}{2\ell}\right)^2$$

$$\left(x+\frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \frac{m^2}{4\ell^2}$$

$$\left(x+\frac{m}{2\ell}\right)^2 = \frac{-4\ell n + m^2}{4\ell^2}$$

$$\left(x+\frac{m}{2\ell}\right)^2 = \frac{m^2 - 4\ell n}{4\ell^2}$$

Taking square root on both sides:-

$$\sqrt{\left(x+\frac{m}{2l}\right)^2} = \pm\sqrt{\frac{m^2-4ln}{4l^2}}$$

$$x+\frac{m}{2l} = \pm\frac{\sqrt{m^2-4ln}}{2l}$$

$$x = -\frac{m}{2l} \pm \frac{\sqrt{m^2-4ln}}{2l}$$

$$x = \frac{-m \pm\sqrt{m^2-4ln}}{2l}$$

Thus solution set $= \left\{\frac{-m \pm\sqrt{m^2-4ln}}{2l}\right\}$

V $\quad 3x^2+7x=0$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

$$3x^2+7x=0$$

$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$

$$x^2+\frac{7}{3}x=0$$

$$x^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking Square root on both sides:-

$$\sqrt{\left(x+\frac{7}{6}\right)^2} = \pm\sqrt{(7/6)^2}$$

$$x+\frac{7}{6} = \pm\frac{7}{6}$$

$$x = \frac{-7}{6} \pm \frac{7}{6}$$

$$x = \frac{-7\pm7}{6}$$

$$x = \frac{-7+7}{6}, \qquad x = \frac{-7-7}{6}$$

$$x = 0 \qquad x = -14/6$$

$$x = -7/3$$

Thus solution set $= \{0, -7/3\}$

vi $x^2-2x-195=0$

$x^2-2x-195=0$

$x^2-2x=195$

$x^2-2(x)(1)+(1)^2=195+(1)^2$

$(x-1)^2=196$

Taking Square root on both sides:-

$\sqrt{(x-1)^2}=\pm\sqrt{196}$

$x-1=\pm 14$

$x=1\pm 14$

$x=1+14$, $x=1-14$

$x=15$, $x=-13$

Thus solution set $=\{-13, 15\}$

vii $-x^2+\frac{15}{2}=\frac{7}{2}x$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

$-x^2+\frac{15}{2}=\frac{7}{2}x$

$x^2+\frac{7}{2}x=+\frac{15}{2}$

$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$

$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$

$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$

$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$

Taking Square root on both sides,

$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$

$x+\frac{7}{4}=\pm\frac{13}{4}$

$x=-\frac{7}{4}\pm\frac{13}{4}$

$x = -\frac{7}{4} + \frac{13}{4}$, $x = -\frac{7}{4} - \frac{13}{4}$

$x = \frac{6}{4}$, $x = \frac{-20}{4}$

$x = 3/2$, $x = -5$

Thus solution set $= \{-5, 3/2\}$

viii $x^2 + 17x + \frac{33}{4} = 0$

$x^2 + 17x + \frac{33}{4} = 0$

$x^2 + 17x = -\frac{33}{4}$

$x^2 + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = -\frac{33}{4} + \left(\frac{17}{2}\right)^2$

$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$

Taking square root on both sides:-

$\sqrt{\left(x + \frac{17}{2}\right)^2} = \pm\sqrt{\frac{256}{4}}$

$x + \frac{17}{2} = \pm\frac{16}{2}$

$x = -\frac{17}{2} \pm \frac{16}{2}$

$x = -\frac{17}{2} + \frac{16}{2}$, $x = -\frac{17}{2} - \frac{16}{2}$

$x = -\frac{1}{2}$, $x = -\frac{33}{2}$

Thus solution set $= \left\{-\frac{1}{2}, -\frac{33}{2}\right\}$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

ix $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

$$4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$\frac{4(3x+1)-8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$\frac{12x+4-8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$\frac{12x-4}{3x+1} = \frac{3x^2+5}{3x+1}$$

Multiplying both sides by $(3x+1)$.

$$12x - 4 = 3x^2 + 5$$

$$3x^2 + 5 - 12x + 4 = 0$$

$$3x^2 - 12x + 9 = 0$$

$$3(x^2 - 4x + 3) = 0$$

$$x^2 - 4x + 3 = 0$$

$$x^2 - 4x = -3$$

$$(x)^2 - (2)(x)(2) + (2)^2 = -3 + (2)^2$$

$$(x-2)^2 = -3 + 4$$

$$(x-2)^2 = 1$$

Taking square root on both sides.

$$x - 2 = \pm 1$$

$$x = 2 \pm 1$$

$x = 2 + 1$, $x = 2 - 1$

$x = 3$ $x = 1$

Thus solution set $= \{3, 1\}$.

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Ex $7(x+2a)^2+3a^2=5a(7x+23a)$

$7(x+2a)^2+3a^2=5a(7x+23a)$

$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$

$7x^2+28ax+28a^2+3a^2=35ax+115a^2$

$7x^2+28ax+28a^2+3a^2-35ax-115a^2=0$

$7x^2-7ax-84a^2=0$

$7(x^2-ax-12a^2)=0$

$x^2-ax-12a^2=0$

$x^2-ax=12a^2$

$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$

$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$

$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$

$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$

Taking square root on both sides:-

$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49}{4}a^2}$

$x-\frac{a}{2}=\pm\frac{7}{2}a$

$x=\frac{a}{2}\pm\frac{7}{2}a$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

$x=\frac{a}{2}+\frac{7}{2}a$, $x=\frac{a}{2}-\frac{7a}{2}$

$x=\frac{8a}{2}$, $x=-\frac{6a}{2}$

$=4a$, $x=-3a$

Thus solution set $=\{-3a, 4a\}$