Ex # 2.7

Solve the following simultaneous equations.

Q.No.1

$x+y=5 \quad ; \quad x^2-2y-14=0$

$x+y=5 \rightarrow (i) \qquad x^2-2y-14=0 \rightarrow (ii)$

From (i)

$x=5-y \rightarrow (A)$

Put in (ii)

$(5-y)^2-2y=14$

$25+y^2-10y-2y=14$

$25+y^2-12y=14$

$y^2-12y+25-14=0$

$y^2-12y+11=0$

$y^2-11y-y+11=0$

$y(y-11)-1(y-11)=0$

$(y-1)(y-11)=0$

$y-1=0 \qquad y-11=0$

$y=1 \qquad y=11$

Put $y=1$ in (A)

$x=5-1$

$x=4$

Put $y=11$ in (A)

$x=5-11$

$x=-6$

So, solution set $\{(4,1),(-6,11)\}$

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Q.No.2

$3x-2y=1$, $x^2+xy-y^2=1$

$3x-2y=1$ — (i) $\quad x^2+xy-y^2=1$ — (ii)

from (i) $3x-2y=1$

$3x=1+2y$

$x=\frac{1+2y}{3}$ — (iii)

put in (ii)

$$\left(\frac{1+2y}{3}\right)^2+\left(\frac{1+2y}{3}\right)y-y^2=1$$

$$\frac{1+4y^2+4y}{9}+\frac{y+2y^2}{3}-y^2=1$$

$$\frac{1+4y^2+4y+3(y+2y^2)-9y^2}{9}=1$$

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$1+4y^2+4y+3y+6y^2-9y^2=9$

$1+7y+10y^2-9y^2=9$

$1+7y+y^2=9$

$y^2+7y-8=0$

$y^2+8y-y-8=0$

$y(y+8)-1(y+8)=0$

$(y-1)(y+8)=0$

$y-1=0$ $\quad y+8=0$

$y=1$ $\quad y=-8$

put $y=-8$ in (iii)

$x=\frac{1+2(-8)}{3}$

$=\frac{1-16}{3}$

$x=\frac{-15}{3}$

$x=-5$

put $y=1$ in (iii)

$x=\frac{1+2(1)}{3}$

$x=\frac{1+2}{3}$

$x=\frac{3}{3}$

$x=1$

So, solution set $\{(-5,-8),(1,1)\}$

Q.No.3

$x-y=7 \quad , \quad \frac{2}{x}-\frac{5}{y}=2$

$x-y=7$ — (i) $\qquad \frac{2}{x}-\frac{5}{y}=2$ — (ii)

From (i) $\quad x=y+7$ — (iii)

put in (ii)

$$\frac{2}{y+7}-\frac{5}{y}=2$$

$$\frac{2y-5(y+7)}{y(y+7)}=2$$

$$2y-5y-35=2y(y+7)$$

$$-3y-35=2y^2+14y$$

$$-3y-35-2y^2-14y=0$$

$$-2y^2-17y-35=0$$

$$-(2y^2+17y+35)=0$$

$$2y^2+7y+10y+35=0$$

$$y(2y+7)+5(2y+7)=0$$

$$(y+5)(2y+7)=0$$

$y+5=0$ $\qquad$ $2y+7=0$

$y=-5$ $\qquad$ $y=\frac{-7}{2}$

Put $y=-5$ in (iii)

$x=-5+7$

$x=2$

put $y=\frac{-7}{2}$ in (iii)

$x=\frac{-7}{2}+7$

$x=\frac{-7+14}{2}$

$x=\frac{7}{2}$

So, solution set $\left\{\left(\frac{7}{2},\frac{-7}{2}\right),(2,-5)\right\}$

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Q.No.4

$x+y=a-b$, $\frac{a}{x}-\frac{b}{y}=2$

$x+y=a-b$ — (i) $\frac{a}{x}-\frac{b}{y}=2$ — (ii)

From (i)

$x=-y+a-b$ — (iii)

put in (ii)

$$\frac{a}{a-b-y}-\frac{b}{y}=2$$

$$\frac{ay-b(a-b-y)}{y(a-b-y)}=2$$

$$ay-ab+b^2+by=2(a-b-y)y$$

$$ay+by+b^2-ab=2ay-2by-2y^2$$

$$ay+by+b^2-ab-2ay+2by+2y^2=0$$

$$-ay+3by+b^2-ab+2y^2=0$$

$$2y^2+3by-ay+b^2-ab=0$$

$$2y^2+(3b-a)y+b^2-ab=0 \quad -(1)$$

Compare (1) by $ax^2+bx+c=0$

and by using quadratic formula:-

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-(3b-a)\pm\sqrt{9b^2+a^2-6ab-8b^2+8ab}}{4}$$

$$=\frac{-(3b-a)\pm\sqrt{b^2+a^2+2ab}}{4}$$

$$=\frac{-(3b-a)\pm\sqrt{(a+b)^2}}{4}$$

$$=\frac{-(3b-a)\pm(a+b)}{4}$$

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$$y = \frac{-(3b-a)+a+b}{4}$$

$$y = \frac{-3b+a+a+b}{4}$$

$$= \frac{2a-2b}{4}$$

$$= \frac{2(a-b)}{4}$$

$$= \frac{a-b}{2}$$

put $y = \frac{a-b}{2}$ in (iii)

$$x = -\frac{a-b}{2} + a - b$$

$$x = \frac{-a+b+2a-2b}{2}$$

$$x = \frac{a-b}{2}$$

$$y = \frac{-(3b-a)-a-b}{4}$$

$$= \frac{-3b+a-a-b}{4}$$

$$= \frac{-4b}{4}$$

$$y = -b$$

put $y = -b$ in (ii)

$$x = -(-b) + a - b$$

$$x = b + a - b$$

$$x = a$$

So solution set is $\{(a,-b), (\frac{a-b}{2}, \frac{a-b}{2})\}$

Q.No.5

$$x^2+(y-1)^2 = 10 \quad , \quad x^2+y^2+4x = 1$$

$$x^2+(y-1)^2 = 10 \text{ — (i)} \qquad x^2+y^2+4x = 1 \text{ — (ii)}$$

$$x^2+y^2+1-2y = 10$$

$$x^2+y^2-2y = 10-1$$

$$x^2+y^2-2y = 9 \text{ — (A)}$$

Subtract (A) and (ii)

$$x^2+y^2-2y = 9$$

$$-x^2 \pm y^2 \pm 4x = \pm 1$$

$$-2y-4x = 8$$

$$-2(y+2x) = 8$$

$$2x+y = -4$$

$$y = -4-2x \text{ — (iii)}$$

put $y = -4-2x$ in (A)

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$$x^2+(-4-2x)^2-2(-4-2x)=9$$

$$x^2+16+4x^2+16x+8+4x=9$$

$$5x^2+20x+24=9$$

$$5x^2+20x=9-24$$

$$5(x^2+4x)=15$$

$$x^2+4x=\frac{15}{5}$$

$$x^2+4x-3=0$$

$$x^2+3x+x+3=0$$

$$x(x+3)+1(x+3)=0$$

$$(x+1)(x+3)=0$$

$x+1=0$ | $x+3=0$

$x=-1$ | $x=-3$

put $x=-1$ in (iii) | put $x=-3$ in (iii)

$y=-4-2(-1)$ | $y=-4-2(-3)$

$y=-4+2$ | $y=-4+6$

$y=-2$ | $y=2$

So solution set $\{(-1,-2),(-3,2)\}$

Q.No.6

$$(x+1)^2+(y+1)^2=5 \quad , \quad (x+2)^2+y^2=5$$

$(x+1)^2+(y+1)^2=5$ — (i) | $(x+2)^2+y^2=5$ — (ii)

$x^2+1+2x+y^2+1+2y=5$ | $x^2+4+2x+y^2=5$

$x^2+y^2+2x+2y=5-1-1$ | $x^2+y^2+4x=5-4$

$x^2+y^2+2x+2y=3$ — (iii) | $x^2+y^2+4x=1$ → (iv)

Subtract (iii) and (iv)

$$x^2+y^2+2x+2y=3$$
$$\pm x^2\pm y^2\pm 4x \quad =\pm 1$$
$$-2x+2y=2$$

$2(-x+y)=2$

$-x+y=1$

$y=1+x$

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$x = y-1$ put in (iii)

$(y-1)^2 + y^2 + 2(y-1) + 2y = 3$

$y^2 + 1 - 2y + y^2 + 2y - 2 + 2y = 3$

$2y^2 + 2y - 4 = 0$

$2(y^2 + y - 2) = 0$

$y^2 + y - 2 = 0$

$y^2 + 2y - y - 2 = 0$

$y(y+2) - 1(y+2) = 0$

$(y-1)(y+2) = 0$

$y - 1 = 0$

$y = 1$

put value of y in (A)

$x = 1 - 1$

$x = 0$

$y + 2 = 0$

$y = -2$

put value of y in (A)

$x = -2 - 1$

$x = -3$

So solution set $\{(-3, -2), (0, 1)\}$

Q. No. 7 $x^2 + 2y^2 = 22$, $5x^2 + y^2 = 29$

$x^2 + 2y^2 = 22$ — (i) $\quad 5x^2 + y^2 = 29$ — (ii)

Multiply eq. (i) with 5 and subtract from (ii)

$5x^2 + 10y^2 = 110$

$\pm 5x^2 \pm y^2 = \pm 29$

$9y^2 = 81$

$y^2 = \frac{81}{9}$

$y^2 = 9$

$y = \pm 3$

Put values of y in (ii)

$5x^2 + (\pm 3)^2 = 29$

$5x^2 + 9 = 29$

$5x^2 = 29 - 9$

$5x^2 = 20$

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$$x^2 = \frac{20}{5}$$

$$x^2 = 4$$

$$x = \pm 2$$

So, solution set is $\{(\pm 2, \pm 3)\}$

Q.No. 8

$4x^2 - 5y^2 = 6$; $3x^2 + y^2 = 14$

$4x^2 - 5y^2 = 6$ — (i) $3x^2 + y^2 = 14$ — (ii)

Multiply (ii) by 5 and add in (i)

$$15x^2 + 5y^2 = 70$$
$$4x^2 - 5y^2 = 6$$
$$\underline{\quad\quad\quad\quad}$$
$$19x^2 = 76$$

$$x^2 = \frac{76}{19}$$

$$x^2 = 4$$

$$x = \pm 2$$

Put $x = \pm 2$ in (ii)

$$3(\pm 2)^2 + y^2 = 14$$

$$3(4) + y^2 = 14$$

$$y^2 = 14 - 12$$

$$y^2 = 2$$

$$y = \pm\sqrt{2}$$

So, solution set is $\{(\pm 2, \pm\sqrt{2})\}$

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Q.No. 9

$7x^2 - 3y^2 = 4$, $2x^2 + 5y^2 = 7$

$7x^2 - 3y^2 = 4$ — (i) $2x^2 + 5y^2 = 7$ — (ii)

Multiply (i) by 5 and (ii) by 3

$35x^2 - 15y^2 = 20$ — (iii) $6x^2 + 15y^2 = 21$ — (iv)

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Add (iii) and (iv)

$$35x^2 + 15y^2 = 20$$
$$6x^2 + 15y^2 = 21$$
$$\overline{\quad\quad\quad\quad\quad\quad}$$
$$41x^2 = 41$$
$$x^2 = 1$$
$$x = \pm 1 \text{ put in (ii)}$$

$$2(\pm 1)^2 + 5y^2 = 7$$
$$2 + 5y^2 = 7$$
$$5y^2 = 7 - 2$$
$$5y^2 = 5$$
$$y^2 = 1$$
$$y = \pm 1$$

So solution set is $\{(\pm 1, \pm 1)\}$

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Q.No. 10

$x^2 + 2y^2 = 3$, $x^2 + 4xy - 5y^2 = 0$

$x^2 + 2y^2 = 3$ — (i) $\quad x^2 + 4xy - 5y^2 = 0$ — (ii)

From eq# (ii)

$$x^2 + 5xy - xy - 5y^2 = 0$$
$$x(x+5y) - y(x+5y) = 0$$
$$(x-y)(x+5y) = 0$$

$x + 5y = 0$ — (3) $\quad x - y = 0$ — (4)

Now attach these equations (3) and (4) with equation (i)

$x^2 + 2y^2 = 3$ — (i)
$x + 5y = 0 \rightarrow (3)$
From (3)
$x = -5y$ — (A)
put in (i)

$x^2 + 2y^2 = 3$ — (i)
$x - y = 0$ — (4)
From Eq. (4)
$x = y$ — (B)
put in (i)

$(-5y)^2 + 2y^2 = 3$

$25y^2 + 2y^2 = 3$

$27y^2 = 3$

$y^2 = \frac{3}{27}$

$y^2 = \frac{1}{9}$

$y = \pm\frac{1}{3}$

$y^2 + 2y^2 = 3$

$3y^2 = 3$

$y^2 = \frac{3}{3}$

$y^2 = 1$

$y = \pm 1$

Put $y = \pm\frac{1}{3}$ in (A)

$x = -5\left(\pm\frac{1}{3}\right)$

$x = \pm\frac{5}{3}$

Put $y = \pm 1$ in (B)

$x = \pm 1$

So, solution set is $\left\{\left(\frac{5}{3}, \frac{1}{3}\right), \left(-\frac{5}{3}, \frac{1}{3}\right), (\pm 1, \pm 1)\right\}$

Q.No.11

$3x^2 - y^2 = 26$, $3x^2 - 5xy - 12y^2 = 0$

$3x^2 - y^2 = 26$ — (1)

$3x^2 - 5xy - 12y^2 = 0$ — (2)

From Eq. (2)

$3x^2 - 9xy + 4xy - 12y^2 = 0$

$3x(x - 3y) + 4y(x - 3y) = 0$

$(3x + 4y)(x - 3y) = 0$

$3x + 4y = 0$ — (3)

$x - 3y = 0$ — (4)

Each equation (3) and (4) with (1)

Now,

$3x^2 - y^2 = 26$ — (1)

$x - 3y = 0$ — (A)

$x = 3y$ — (A)

Put $x = 3y$ in (1)

$3(3y)^2 - y^2 = 26$

$27y^2 - y^2 = 1026$

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$$26y^2 = 26$$

$$y^2 = 1$$

$$y = \pm 1 \quad \text{put in (A)}$$

$$x = \pm 3$$

Now,

$$3x^2 - y^2 = 26 \quad \text{— (1)}$$

$$3x + 4y = 0 \quad \text{— (3)}$$

From (3)

$$x = -\frac{4}{3}y \rightarrow \text{(B)}$$

put in (1)

$$3\left(-\frac{4}{3}y\right)^2 - y^2 = 26$$

$$3\left(\frac{16}{9}y^2\right) - y^2 = 26$$

$$\frac{16}{3}y^2 - y^2 = 26$$

$$16y^2 - 3y^2 = 78$$

$$13y^2 = 78$$

$$y^2 = \frac{78}{13}6$$

$$y = \pm\sqrt{6}$$

put in (B)

$$x = -\frac{4}{3}(\pm\sqrt{6})$$

$$x = \pm\frac{4\sqrt{6}}{3}$$

So, solution set is $\{(3,1)(-3,-1), (\frac{-4\sqrt{6}}{3}, \sqrt{6})$

$$\left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\}$$

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Q.NO.12

$x^2+xy=5 \qquad y^2+xy=3$

$x^2+xy=5$ — (1) $\qquad y^2+xy=3$ — (2)

Multiply eq.(1) by (3) and eq(2) by 5 then subtract eq(2) from eq(1).

$$3x^2+3xy=15$$
$$\pm 5y^2 \pm 5xy = \pm 15$$
$$3x^2-5y^2-2xy=0$$
$$3x^2-5xy+3xy-5y^2=0$$
$$x(3x-5y)+y(3x-5y)=0$$
$$(x+y)(3x-5y)=0$$

$x+y=0$

$y=-x$

Put $x=-x$ in eq(i)

$x^2+x(-x)=5$

$x^2-x^2=5$

$0=5$

Not possible

$3x-5y=0$

$3x=5y$

$x=\frac{5}{3}y$

Put $x=\frac{5}{3}y$ in eq(ii)

$(\frac{5}{3}y)^2+(\frac{5}{3}y)y=5$

$\frac{25}{9}y^2+\frac{5}{3}y^2=5$

$\frac{25y^2+15y^2}{9}=5$

$40y^2=45$

$y^2=\frac{45}{40}$

$y=\pm\sqrt{\frac{9}{8}}$

$y=\pm\frac{3}{2\sqrt{2}}$

Put $y=\pm\frac{3}{2\sqrt{2}}$ in $x=\frac{5}{3}y$

$x=\frac{5}{3}(\pm\frac{3}{2\sqrt{2}})$

$x=\pm\frac{5}{2\sqrt{2}}$

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Thus solution set is $\left\{\left(\frac{5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}}\right), \left(-\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right)\right\}$

Q.NO. 13

$x^2 - 2xy = 7$, $xy + 3y^2 = 2$

$x^2 - 2xy = 7$ — (i) $\quad xy + 3y^2 = 2$ — (ii)

Multiply eq (i) by 2 and eq (ii) by 7 then Subtract eq (ii) from eq. (i)

$$2x^2 - 4xy = 14$$
$$\pm 21y^2 \pm 7xy = \pm 14$$

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(2x + 3y)(x - 7y) = 0$$

$2x + 3y = 0$

$2x = -3y$

$x = -\frac{3}{2}y$ — (A)

$x - 7y = 0$

$x = 7y$ — (B)

Put $x = -\frac{3}{2}y$ in eq (ii)

$x = -\frac{3}{2}y$

$\left(-\frac{3y}{2}\right)y + 3y^2 = 2$

$-\frac{3y^2}{2} + 3y^2 = 2$

$-3y^2 + 6y^2 = 4$

$3y^2 = 4$

$y^2 = \frac{4}{3}$

$y = \frac{\pm 2}{\sqrt{3}}$

Put value in A

$x = \frac{-3}{2}\left(\pm \frac{2}{\sqrt{3}}\right)$

Put $x = 7y$ in eq (ii)

$(7y)y + 3y^2 = 2$

$7y^2 + 3y^2 = 2$

$10y^2 = 2$

$y^2 = \frac{2}{10}$

$y = \pm\frac{1}{\sqrt{5}}$

Put in B

$x = 7\left(\pm\frac{1}{\sqrt{5}}\right)$

$x = \pm\frac{7}{\sqrt{5}}$

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$$x = \pm \frac{3}{\sqrt{3}}$$

$$x = \pm\sqrt{3}$$

So solution set $\left\{ \left(\pm\sqrt{3}, \pm\frac{2}{\sqrt{3}}\right) \left(\pm\frac{7}{\sqrt{5}}, \pm\frac{1}{\sqrt{5}}\right) \right\}$

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