Ex # 2.5

Q.No.1

Write the quadratic equations having following roots.

(a) 1, 5

Since 1, 5 are the roots of required equation.

So, S = Sum of roots = $1+5$

$S = 6$

P = product of roots = 1×5

$P = 5$

As $x^2 - Sx + P = 0$, so the required Equation is $x^2 - 6x + 5 = 0$

(b) 4, 9

Since, 4, 9 are the roots of required equation.

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So, $S = 4 + 9 = 13$

$P = 4 \times 9 = 36$

As $x^2 - Sx + P = 0$

So, $x^2 - 13x + 36 = 0$

(c) −2, 3

Since, −2, 3 are the roots of required Equation.

So, $S = -2 + 3 = 1$

$P = -2 \times 3 = -6$

Thus, the required Equation is

$x^2 - Sx + P = 0$

$x^2 + x - 6 = 0$

(d) $0, -3$

Since $0, -3$ are the roots of required Equation. Therefore,

$$S = Sum = 0 - 3 = -3$$

$$P = 0 \times 3 = 0$$

Thus, the required Equation,

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 0 = 0$$

$$x^2 + 3x = 0$$

(e) $2, -6$

Since $2, -6$ are the roots of Equation.

So, $S = 2 - 6 = -4$

$$P = (2)(-6) = -12$$

Thus, required equation is

$$x^2 - Sx + P = 0$$

$$x^2 + 4x - 12 = 0$$

(f) $1 + i, 1 - i$

Since $(1+i), (1-i)$ are the roots of Equation.

So, $S = 1 + i + 1 - i = 2$

$$P = (1+i)(1-i) = 1 - i^2$$

$$P = 1 - (-1) = 2$$

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Thus, required Equation is,

$$x^2 - Sx + P = 0$$

$$x^2 - 2x + 2 = 0$$

(h) $(3 + \sqrt{2}), (3 - \sqrt{2})$

Since $(3+\sqrt{2}), (3-\sqrt{2})$ are the roots of required Equation.

Therefore, $S = 3 + \sqrt{2} + 3 - \sqrt{2}$

$$S = 6$$

$P = (3+\sqrt{2})(3-\sqrt{2})$

$= 9 - (\sqrt{2})^2$

$P = 7$

Thus, required equation is

$x^2 - Sx + P = 0$

$x^2 - 6x + 7 = 0$

Q.No.2

If α, β are the roots of the equation $x^2 - 3x + 6 = 0$ From equations whose roots are

(a)

$2\alpha + 1, \ 2\beta + 1$

As α, β are the roots of given Eq.

$x^2 - 3x + 6 = 0$

Sum of roots $= \alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = 3$

Product of roots $= \alpha\beta = \frac{c}{a}$

$\alpha\beta = 6$

As $2\alpha + 1$ and $2\beta + 1$ are the roots of required equation.

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$S = 2\alpha + 1 + 2\beta + 1$

$= 2(\alpha + \beta) + 2$

$S = 2(3) + 2 = 8$

Product of roots $= (2\alpha + 1)(2\beta + 1)$

$P = 4\alpha\beta + 2\alpha + 2\beta + 1$

$= 4\alpha\beta + 2(\alpha + \beta) + 1$

$= 4(6) + 2(3) + 1$

$= 24 + 6 + 1$

$P = 31$

So required equation:-

$x^2 - Sx + P = 0$

$x^2 - 8x + 31 = 0$

b) α^2, β^2

As $\alpha+\beta = 3$, $\alpha\beta = 6$

Since α^2 and β^2 are the roots of required Equation,

So, $\text{Sum} = \alpha^2+\beta^2$

$S = (\alpha+\beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(6)$

$= 9 - 12$

$S = -3$

$\text{product} = (\alpha^2)(\beta^2)$

$P = (\alpha\beta)^2$

$P = (6)^2$

$P = 36$

So, the required equation is,

$x^2 - Sx + P = 0$

$x^2 - (-3)x + 36 = 0$

$x^2 + 3x + 36 = 0$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

As $\alpha+\beta = 3$, $\alpha\beta = 6$

Since $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of Equation.

So, $\text{Sum} = \frac{1}{\alpha} + \frac{1}{\beta}$

$S = \frac{\alpha+\beta}{\alpha\beta}$

$S = \frac{3}{6}$

$S = \frac{1}{2}$

$\text{product} = \frac{1}{\alpha} \cdot \frac{1}{\beta}$

$P = \frac{1}{\alpha\beta}$

$P = \frac{1}{6}$

So required Equation

$x^2 - Sx + P = 0$

$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$ or $6x^2 - 3x + 1 = 0$

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(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

As $\alpha+\beta = 3$, $\alpha\beta = 6$

Since $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of Equation.

So, Sum $= \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$S = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(3)^2 - 2(6)}{6}$

$= \frac{9-12}{6} = -\frac{3}{6}$

$S = -\frac{1}{2}$

Product $= \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha}$

$P = 1$

So, the required equation is

$x^2 - Sx + P = 0$

$x^2 - \left(\frac{-1}{2}\right)x + 1 = 0$

$2x^2 + x + 2 = 0$

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(e) $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta}$

As $\alpha+\beta = 3$ and $\alpha\beta = 6$

Since $\alpha+\beta$ and $\frac{1}{\alpha}+\frac{1}{\beta}$ are the roots of required equation.

So, sum $= \alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}$

$S = \frac{\alpha^2\beta + \alpha\beta^2 + \beta + \alpha}{\alpha\beta}$

$S = \frac{\alpha\beta(\alpha+\beta) + (\alpha+\beta)}{\alpha\beta} = \frac{6(3)+3}{6}$

$$S = \frac{18+3}{6} = \frac{21}{6}$$

$$S = \frac{7}{2}$$

and the product $= (\alpha+\beta)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$$= (\alpha+\beta)\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(3)^2}{6} = \frac{9}{6}$$

$$P = \frac{3}{2}$$

So the required Equation

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$2x^2 - 7x + 3 = 0$$

Q.No.03

If α, β are the roots of the equation $x^2+px+q=0$ Form equation whose roots are.

(a) α^2, β^2

$$x^2 + px + q = 0$$

So, $\alpha+\beta = \frac{-b}{a}$

$$\alpha+\beta = -p$$

and $\alpha\beta = \frac{c}{a} = q$

As α^2 and β^2 are the roots of equation

So, Sum $= \alpha^2 + \beta^2$

$$S = (\alpha+\beta)^2 - 2\alpha\beta$$

$$S = p^2 - 2q$$

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and $\quad$ product $= \alpha^2\beta^2$

$$P = (\alpha\beta)^2$$

$$P = q^2$$

So, the required Equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (p^2 - 2q)x + q^2 = 0$$

(b)

$$\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$$

As $\alpha + \beta = -p$ and $\alpha\beta = q$

Since $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are the roots of required equation.

So, sum $= \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(-p)^2 - 2(q)}{q}$$

$$S = \frac{p^2 - 2q}{q}$$

and $\quad$ product $= \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)$

$$P = 1$$

So, required Equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

$$qx^2 - (p^2 - 2q)x + q = 0$$