Ex # 2.3

Q # 1

Without solving, find the sum and the product of the roots of the following quadratic Equations.

i $$x^2 - 5x + 3 = 0$$

Here, $a = 1$, $b = -5$, $c = 3$

Let, α and β be the roots of given equation.

Then sum of roots $= \alpha + \beta = \frac{-b}{a}$

$$= \frac{-(-5)}{1} = 5$$

And the product of roots $= \alpha\beta = \frac{c}{a}$

$$= \frac{+3}{1} = 3$$

ii $$3x^2 + 7x - 11 = 0$$

Here $a = 3$, $b = 7$, $c = -11$

α and β be the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = \frac{-7}{3}$

product of roots $= \alpha\beta = \frac{c}{a} = \frac{-11}{3}$

iii $$px^2 - qx + r = 0$$

Here, $a = p$, $b = -q$, $c = r$

Let α and β be the roots of equation.

So, sum of roots $= \alpha + \beta = -\frac{b}{a} = \frac{q}{p}$

product of roots $= \alpha\beta = \frac{c}{a} = \frac{r}{p}$

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Ex # 2.3 Q # 1

iv $(a+b)x^2 - ax + b = 0$

Here, $a = a+b$, $b = -a$, $c = b$

Let α and β are the roots of equation.

So, sum of roots $= \alpha + \beta = \frac{-b}{a} = \frac{a}{a+b}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{b}{a+b}$

v $(l+m)x^2 + (m+n)x + n - l = 0$

Here, $a = l+m$, $b = m+n$, $c = n-l$

Let α and β are the roots of equation

So, sum of roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(m+n)}{l+m}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{n-l}{l+m}$

vi $7x^2 - 5mx + 9n = 0$

Here, $a = 7$, $b = -5m$, $c = 9n$

Let α and β are the roots of Equation.

So, sum of roots $= \alpha + \beta = \frac{-b}{a}$

$= \frac{5m}{7}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{9n}{7}$

Q.No.2

Find the value of k, if.

i Sum of roots of the equation $2kx^2 - 3x + 4k = 0$ is twice the product of the roots.

$2kx^2 - 3x + 4k = 0$

Here, $a = 2k$, $b = -3$, $c = 4k$

Let α and β are the roots of equation.

So, sum of roots $= \alpha + \beta = \frac{-b}{a} = \frac{3}{2k}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{4k}{2k}$

$= 2$

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As sum of the roots is twice the product of roots. so,

$$\alpha+\beta = 2(\alpha\beta)$$

$$\frac{3}{2K} = 2(2)$$

$$\frac{3}{2K} = 4$$

$$\frac{1}{K} = \frac{8}{3}$$

$$K = \frac{3}{8}$$

ii Sum of the roots of equation $x^2+(3K-7)x+5K=0$ is $\frac{3}{2}$ times the product of the roots.

$$x^2+(3K-7)x+5K=0$$

Here, $a=1$, $b=3K-7$, $c=5$

Let α and β are the roots of equation.

So, sum of roots $= \alpha+\beta = \frac{-b}{a} = -3K+7$

Product of roots $= \alpha\beta = \frac{c}{a} = 5K$

As sum of the roots is $\frac{3}{2}$ times of the product of roots. so,

$$\alpha+\beta = \frac{3}{2}(\alpha\beta)$$

$$-3K+7 = \frac{3}{2}(5K)$$

$$2(-3K+7) = 3(5K)$$

$$-6K+14 = 15K$$

$$15K+6K = 14$$

$$21K = 14$$

$$K = \frac{14}{21}$$

$$K = \frac{2}{3}$$

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Q.No.3

Find K, if

i Sum of the squares of the roots of the equation.

$$4Kx^2+3Kx-8=0 \text{ is } 2.$$

$$4Kx^2+3Kx-8=0$$

Here, $a=4K$, $b=3K$, $c=-8$

Let α and β be the roots the given equation

so, $\text{sum of roots} = \frac{-b}{a} = -\frac{3K}{4K}$

$$\alpha+\beta = -\frac{3}{4}$$

$$\text{product of roots} = \frac{c}{a} = -\frac{8}{4K}$$

$$\alpha\beta = -\frac{2}{K}$$

As given that:-

$$\alpha^2+\beta^2 = 2 \longrightarrow (i)$$

since $(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$$

So Eq (i) becomes

$$(\alpha+\beta)^2-2\alpha\beta = 2$$

$$\left(-\frac{3}{4}\right)^2 - 2\left(-\frac{2}{K}\right) = 2$$

$$\frac{9}{16}+\frac{4}{K} = 2$$

$$\frac{4}{K} = 2-\frac{9}{16}$$

$$\frac{4}{K} = \frac{23}{16}$$

$$23K = 64$$

$$K = \frac{64}{23}$$

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ii Sum of the squares of roots of the equation $x^2-2kx+(2k+1)=0$ is "6".

$$x^2-2kx+(2k+1)=0$$

Here $a=1$, $b=-2k$, $c=2k+1$

Let α and β are the roots of equation.

So, sum of roots $=\alpha+\beta=\frac{-b}{a}$

$$\alpha+\beta=2k$$

$$\text{product of roots}=\alpha\beta=\frac{c}{a}$$

$$\alpha\beta=2k+1$$

As, Sum of square of roots are "6".

$$\alpha^2+\beta^2=6$$

$$\because (\alpha+\beta)^2-2\alpha\beta=\alpha^2+\beta^2$$

$$\text{so, } (\alpha+\beta)^2-2\alpha\beta=6$$

$$(2k)^2-2(2k+1)=6$$

$$4k^2-4k-2=6$$

$$4k^2-4k-2-6=0$$

$$4k^2-4k-8=0$$

$$4(k^2-k-2)=0$$

$$k^2-k-2=0$$

$$k^2-2k+k-2=0$$

$$k(k-2)+1(k-2)=0$$

$$(k+1)(k-2)=0$$

Either $k+1=0$ or $k-2=0$

$k=-1$ $\qquad$ $k=2$

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Q. No. 4

Find P, if

i The roots of the equation $x^2-x+P^2=0$ differ by unity.

$$x^2-x+P^2=0$$

Here $a=1$, $b=-1$, $c=P^2$

Let α and $\alpha-1$ are the roots of equation.

then $\text{sum} = \alpha+\alpha-1 = \frac{-b}{a}$

$$\alpha+\alpha-1 = 1$$

$$2\alpha=2 \Rightarrow \alpha=1$$

$$\text{product} = \alpha\beta = \frac{c}{a}$$

$$\alpha(\alpha-1) = P^2$$

$$\alpha^2-\alpha = P^2$$

$$(1)^2-1 = P^2$$

$$0 = P^2$$

$$\text{OR } P=0$$

ii The roots of the equation $x^2+3x+P-2=0$ differ by 2.

$$x^2+3x+P-2=0$$

Here $a=1$, $b=3$, $c=P-2$

Let α and $\alpha-2$ be the roots of equation.

Then, $\text{Sum} = \alpha+\alpha-2 = \frac{-b}{a}$

$$2\alpha-2=-3$$

$$2\alpha=-3+2$$

$$\alpha=-\frac{1}{2}$$

$$\text{Product} = \alpha(\alpha-2) = \frac{c}{a}$$

$$\alpha^2-2\alpha = P-2$$

$$\left(-\frac{1}{2}\right)^2-2(-1/2) = P-2$$

$$\frac{1}{4} + 1 = p - 2$$

$$\frac{1}{4} + 1 + 2 = p$$

$$\frac{1}{4} + 3 = p$$

$$\frac{1+12}{4} = p$$

OR $p = \frac{13}{4}$

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Q. No. 5

Find m, if

i the roots of the equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$

$$x^2 - 7x + 3m - 5 = 0$$

Here, $a = 1$, $b = -7$, $c = 3m - 5$

Let α and β be the roots of equation.

then sum $= \alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = 7 \Rightarrow \beta = 7 - \alpha \rightarrow (i)$$

Product $= \alpha\beta = \frac{c}{a}$

$$\alpha\beta = 3m - 5 \text{ —— (ii)}$$

Since $3\alpha + 2\beta = 4$ — (iii)

put β in eq (iii):

$$3\alpha + 2(7 - \alpha) = 4$$

$$3\alpha + 14 - 2\alpha = 4$$

$$3\alpha - 2\alpha = 4 - 14$$

$\alpha = -10$ put in (i)

$$\beta = 7 - (-10)$$

$$\beta = 17$$

put α and β in Eq. (ii)

$$(-10)(17) = 3m - 5$$

$$-170 + 5 = 3m$$

$$m = \frac{-165}{3} \Rightarrow m = -55$$

ii The roots of the equation $x^2+7x+3m-5=0$ satisfy relation $3\alpha-2\beta=4$.

Given that:- $3\alpha-2\beta=4$ —— (i)

$x^2+7x+3m-5=0$

Here, $a=1$, $b=7$, $c=3m-5$

Let α and β are the roots.

So $\alpha+\beta=-7 \Rightarrow \beta=-7-\alpha \rightarrow$ (ii)

$\alpha\beta=3m-5$ —— (iii)

Put β in Eq. (i)

$3\alpha-2(-7-\alpha)=4$

$3\alpha+14+2\alpha=4$

$5\alpha=-10$

$\alpha=-2$

Put α in Eq. (ii)

$\beta=-7-(-2)$

$\beta=-5$

Put α and β in (iii)

$(-2)(-5)=3m-5$

$10+5=3m$

$3m=15$

$m=3$

iii The roots of the equation $3x^2-2x+7m+2=0$ satisfy relation $7\alpha-3\beta=18$

$3x^2-2x+7m+2=0$

Here $a=3$, $b=-2$, $c=7m+2$

Let α and β are the roots

$\alpha+\beta=-\frac{(-2)}{3}$

$\alpha+\beta=\frac{2}{3}$

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$$\beta = \frac{2}{3} - \alpha \quad \text{—— (i)}$$

product of roots $= \frac{7m+2}{3}$

$$\alpha\beta = \frac{7m+2}{3} \quad \text{—— (ii)}$$

since $\quad 7\alpha - 3\beta = 18 \quad \text{—— (iii)}$

put $\beta = \frac{2}{3} - \alpha$ in eq. (iii)

$$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$$

$$7\alpha - 2 + 3\alpha = 18$$

$$10\alpha = 18 + 2$$

$$\alpha = \frac{20}{10}$$

$$\alpha = 2$$

put $\alpha = 2$ in eq (i)

$$\beta = \frac{2}{3} - 2$$

$$\beta = -\frac{4}{3}$$

put $\alpha = 2$ and $\beta = -\frac{4}{3}$ in eq (ii)

$$(2)\left(-\frac{4}{3}\right) = \frac{7m+2}{3}$$

$$-\frac{8}{3} = \frac{7m+2}{3}$$

$$-\frac{8}{3} \times 3 = 7m + 2$$

$$7m = -8 - 2$$

$$m = \frac{-10}{7}$$

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Q.No.6

Find m, if sum and product of the roots of the following equations is equal to a given number λ.

i $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Here $a = 2m+3$, $b = 7m-5$, $c = 3m-10$

Let α and β are the roots of equation

$$\alpha + \beta = \frac{-b}{a} = \frac{-(7m-5)}{2m+3}$$

$$\alpha\beta = \frac{3m-10}{2m+3}$$

As $(\alpha+\beta) = \lambda$ and $\alpha\beta = \lambda$

So, By comparing,

$$\alpha + \beta = \alpha\beta$$

$$-\frac{(7m-5)}{2m+3} = \frac{3m-10}{2m+3}$$

$$-(7m-5)(2m+3) = (3m-10)(2m+3)$$

$$-(14m^2 + 21m - 10m - 15) = 6m^2 - 20m + 9m - 30$$

$$-(14m^2 + 11m - 15) = 6m^2 - 11m - 30$$

$$-14m^2 - 11m + 15 - 6m^2 + 11m + 30 = 0$$

$$-20m^2 + 45 = 0$$

$$20m^2 = 45$$

$$m^2 = \frac{45}{20}$$

$$m^2 = \frac{9}{4}$$

$$m = \frac{3}{2}$$

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ii $4x^2 - (3+5m)x - (9m-17) = 0$

$$4x^2 - (3+5m)x - (9m-17) = 0$$

Here $a = 4$, $b = -(3+5m)$, $c = -(9m-17)$

Let α and β are the roots.

Then $\alpha + \beta = -\dfrac{b}{a}$

$$\alpha + \beta = \frac{3+5m}{4}$$

and $\alpha\beta = \dfrac{c}{a} = -\dfrac{(9m-17)}{4}$

Since $\alpha + \beta = r$ and $\alpha\beta = \lambda$

So, $\alpha + \beta = \alpha\beta$

$$\frac{3+5m}{4} = -\frac{(9m-17)}{4}$$

$$3 + 5m = -9m + 17$$

$$5m + 9m = 17 - 3$$

$$14m = 14$$

$$m = 1$$

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