Ex # 2.2

Q.No.1

Find the cube roots of $-1, 8, -27, 64$.

i Cube roots of -1

Let $x^3 = -1$

$$x^3 + (1)^3 = 0$$

$$(x+1)(x^2 - x + 1) = 0$$

Either $x + 1 = 0$ or $x^2 - x + 1 = 0$

$x = -1$

Here $a = 1,\ b = -1,\ c = 1$

Using quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 + \sqrt{-3}}{2}, \quad x = \frac{1 - \sqrt{-3}}{2}$$

$$= -\omega^2 \qquad = -\omega$$

three cube roots of -1 are $-1, -\omega, -\omega^2$

ii Cube roots of 8

Let $x^3 = 8$

$$x^3 - 8 = 0$$

$$(x)^3 - (2)^3 = 0$$

$$(x-2)(x^2 + 2x + 4) = 0$$

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$

$x = 2$

By using quadratic formula:-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

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$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$= \frac{\cancel{2}(-1 \pm \sqrt{-3})}{\cancel{2}}$$

$$= -1 \pm i\sqrt{3}$$

$$x = -1 + i\sqrt{3}, \quad x = -1 - i\sqrt{3}$$

$$x = 2\omega, \quad x = 2\omega^2$$

Three cube roots of 8 are $2, 2\omega, 2\omega^2$.

iii Cube roots of -27

Let $x^3 = -27$

$$x^3 + 27 = 0$$

$$(x)^3 + (3)^2 = 0$$

$$(x+3)(x^2 - 3x + 9) = 0$$

Either $x + 3 = 0$ or $x^2 - 3x + 9 = 0$

$x = -3$ using quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-36}}{2}$$

$$= \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3 \pm 3\sqrt{-3}}{2}$$

$$= 3\left(\frac{1 \pm \sqrt{-3}}{2}\right)$$

$$x = 3\left(\frac{1 + \sqrt{-3}}{2}\right), \quad x = 3\left(\frac{1 - \sqrt{-3}}{2}\right)$$

$$x = -3\omega \qquad x = -3\omega^2$$

Three cube roots 27 are $-3, -3\omega, -3\omega^2$.

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iv Cube root of 64

Let $x^3 = 64$

$$x^3 - (4)^3 = 0$$

$$(x-4)(x^2+4x+16) = 0$$

$x - 4 = 0$

$x = 4$

$x^2 + 4x + 16 = 0$

using quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$= \frac{-4 \pm 4\sqrt{3}}{2}$$

$$= 4\left(\frac{-1 \pm i\sqrt{3}}{2}\right)$$

$x = 4\left(\frac{-1 + i\sqrt{3}}{2}\right)$, $x = 4\left(\frac{-1 - i\sqrt{3}}{2}\right)$

$x = 4w$ $\quad$ $x = 4w^2$

Three cube roots of 64 is $4, 4w, 4w^2$

Q. No. 2

Evaluate

i $(1-w-w^2)^7$

$$(1-w-w^2)^7 = [1-(w+w^2)]^7$$

$$= [1-(-1)]^7$$

$$= 2^7 \Rightarrow 128$$

ii $(1-3w-3w^2)^5$

$$(1-3w-3w^2)^5 = [1-3(w+w^2)]^5$$

$$= [1-3(-1)]^5$$

$$= (4)^5$$

$$= 1024$$

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iii $(9+4w+4w^2)^3$

$$(9+4w+4w^2)^3 = [9+4(w+w^2)]^3$$
$$= [9+4(-1)]^3$$
$$= [9-4]^3$$
$$= 5^3 \Rightarrow 125$$

iv $(2+2w-2w^2)(3-3w+3w^2)$

$$(2+2w-2w^2)(3-3w+3w^2)$$
$$= [2(1+w)-2w^2][3(1+w^2)-3w]$$
$$= [2(-w^2)-2w^2][3(-w)-3w]$$
$$= (-2w^2-2w^2)(-3w-3w)$$
$$= (-4w^2)(-6w)$$
$$= 24w^3 \qquad \because w^3 = 1$$
$$= 24$$

v $(-1+\sqrt{-3})^6+(-1-\sqrt{3})^6$

$$= (-1+\sqrt{-3})^6 + (-1-\sqrt{3})^6$$
$$= (2w)^6 + (2w^2)^6$$
$$= 2^6[(w^3)^2+(w^3)^2]$$
$$= 2^6[(1)^2+(1)^2]$$
$$= 2^6 \cdot 2$$
$$= 128$$

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vi $\left(\frac{-1+\sqrt{-3}}{2}\right)^9+\left(\frac{-1-\sqrt{-3}}{2}\right)^9$

$$= w^9+(w^2)^9 \qquad \because w = \frac{-1+\sqrt{-3}}{2}$$
$$= w^9+w^{18} \qquad \because w^2 = \frac{-1-\sqrt{-3}}{2}$$
$$= (w^3)^3+(w^3)^6$$
$$= 1+1 = 2$$

vii

$$\omega^{-13} + \omega^{-17}$$

$$\omega^{-13} + \omega^{-17}$$

$$= \omega^{-12}\cdot\omega^{-1} + \omega^{-15}\cdot\omega^{-2}$$

$$= (\omega^3)^{-4}\cdot\omega^{-1} + (\omega^3)^{-5}\cdot\omega^{-2}$$

$$= (1)^{-4}\cdot\omega^{-1} + (1)^{-5}\cdot\omega^{-2}$$

$$= \omega^{-1} + \omega^{-2}$$

$$= \frac{1}{\omega^{1}} + \frac{1}{\omega^2}$$

$$= \frac{\omega^2 + \omega}{\omega^3}$$

$$= \frac{-1}{1} = -1$$

Q.No.3

Prove that:-

$$x^3 + y^3 = (x+y)(x+\omega y)(x+\omega^2 y)$$

$$x^3 + y^3 = (x+y)(x+\omega y)(x+\omega^2 y)$$

R.H.S

$$= (x+y)(x+\omega y)(x+\omega^2 y)$$

$$= (x+y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2)$$

$$= (x+y)[x^2 + (\omega^2+\omega)xy + (1)y^2]$$

$$= (x+y)[x^2 + (-1)xy + y^2]$$

$$= (x+y)(x^2 - xy + y^2)$$

$$= x^3 + y^3$$

Hence proved.

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Q.No.4

Prove that

$$x^3+y^3+z^3-3xyz=(x+y+z)(x+wy+w^2z)(x+w^2y+wz)$$

$$x^3+y^3+z^3-3xyz=(x+y+z)(x+wy+w^2z)(x+w^2y+wz)$$

R.H.S $= (x+y+z)(x+wy+w^2z)(x+w^2y+wz)$

$= (x+y+z)[x(x+w^2y+wz)+wy(x+w^2y+wz)+w^2z(x+w^2y+wz)]$

$= (x+y+z)[x^2+w^2xy+wxz+wxy+w^3y^2+w^2yz+w^2xz+w^3yz+w^3z^2]$

$= (x+y+z)[x^2+(w^2+w)xy+(w^2+w)yz+(w^2+w)xz+y^2+z^2]$

$= (x+y+z)[x^2+(-1)xy+(-1)yz+(-1)xz+y^2+z^2]$

$= (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= x^3+y^3+z^3-3xz$

$=$ L.H.S

Hence proved.

Q.No.5

Prove that

$$(1+w)(1+w^2)(1+w^4)(1+w^8)\ldots 2n \text{ factors}=1.$$

$(1+w)(1+w^2)(1+w^4)(1+w^8)\ldots 2n$ factors $=1$

L.H.S $= (1+w)(1+w^2)(1+w^4)(1+w^8)\ldots 2n$ factors.

$= (1+w)(1+w^2)(1+w^3\cdot w)(1+w^2\cdot(w^3)^2)\ldots 2n$ factors.

$= (1+w)(1+w^2)(1+w)(1+w^2)\ldots 2n$ factors $\quad \because w^3=1$

$= (-w)^2(-w)(-w^2)(-w)\ldots 2n$ factors.

$= (w^3)(w^3)\ldots 2n$ factors.

$= (1)(1)\ldots n$ factors.

$= (1)^n$

$= 1$

$=$ R.H.S

Hence proved.

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