CH·No.2

Theory Of Quadratic Equations

Ex#2.1

Q.No.1

Find the discriminant of the following given quadratic equations:

i $2x^2+3x-1=0$

$2x^2+3x-1=0$

compare it with

$ax^2+bx+c=0$

Here $a=2$, $b=3$, $c=1$

Disc. $=b^2-4ac$

$=(3)^2-4(2)(-1)$

$=9+8$

$=17$

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ii $6x^2-8x+3=0$

$6x^2-8x+3=0$

Here $a=6$, $b=-8$, $c=3$

Disc. $=b^2-4ac$

$=(-8)^2-4(6)(3)$

$=64-72$

$=-8$

iii $9x^2-30x+25=0$

Here, $a=9$, $b=-30$, $c=25$

Disc $=b^2-4ac$

$=(-30)^2-4(9)(25)$

$=900-900=0$

iv $$4x^2-7x-2=0$$

Here, $a=4,\ b=-7,\ c=-2$

$$\text{Disc.}=b^2-4ac$$
$$=(-7)^2-4(4)(-2)$$
$$=49+32$$
$$=81$$

Q.No.2

Find the nature of the roots of the following given equations and verify the result by solving the equations:

i $$x^2-23x+120=0$$

Here $a=1,\ b=-23,\ c=120$

$$\text{Disc.}=b^2-4ac$$
$$=(-23)^2-4(1)(120)$$
$$=529-480$$
$$=49$$
$$=(7)^2>0$$

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As the disc. is positive and is perfect square. Therefore the roots are rational and unequal.

Verification by solving the equation.

$$x^2-23x+120=0$$
$$x^2-15x-8x+120=0$$
$$x(x-15)-8(x-15)=0$$
$$(x-8)(x-15)=0$$

Either $x-8=0$ or $x-15=0$

$x=8$ $\qquad$ $x=15$

Thus roots are rational and unequal.

ii $2x^2+3x+7=0$

Here $a=2,\ b=3,\ c=7$

$$\begin{aligned}\text{Disc.} &= b^2-4ac\\ &=(3)^2-4(2)(7)\\ &=9-56\\ &=-47<0\end{aligned}$$

As the disc. is negative

Therefore, roots are imaginery and unequal.

$\Rightarrow$ Verification by solving equation:-

$2x^2+3x+7=0$

Using quadratic formula

$$\begin{aligned}x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\ &=\frac{-3\pm\sqrt{(3)^2-4(2)(7)}}{2(2)}\\ &=\frac{-3\pm\sqrt{9-56}}{4}=\frac{-3\pm\sqrt{47}}{4}\end{aligned}$$

Thus the roots are imaginary and unequal.

iii $16x^2-24x+9=0$

Here $a=16,\ b=-24,\ c=9$

$$\begin{aligned}\text{Disc.} &= b^2-4ac\\ &=(-24)^2-4(16)(9)\\ &=576-576\\ &=0\end{aligned}$$

As disc. is zero.

Therefore, the roots of the equation are real and equal.

Verification:-

$$\begin{aligned}16x^2-24x+9&=0\\ (4x)^2-2(4x)(3)+(3)^2&=0\\ (4x-3)^2&=0\end{aligned}$$

$$4x - 3 = 0$$
$$4x = 3$$
$$x = 3/4$$

Thus roots are real.

iv $$3x^2 + 7x - 13 = 0$$

Here, $a = 3$, $b = 7$, $c = -13$

$$\text{Disc.} = b^2 - 4ac$$
$$= (7)^2 - 4(3)(-13)$$
$$= 49 + 156$$
$$= 205 > 0$$

As disc. is positive and not a perfect square.

Therefore, the roots are real and unequal.

Varification:-

$$3x^2 + 7x - 13 = 0$$

using quadratic formula:-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{49 - 4(3)(-13)}}{2(3)}$$

$$= \frac{-7 \pm \sqrt{49 + 156}}{6}$$

$$= \frac{-7 \pm \sqrt{205}}{6}$$

Thus, the roots are real and unequal.

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Q.No.3

For what value of k, the expression $k^2x^2+2(k+1)x+4$ is perfect square.

$$k^2x^2+2(k+1)x+4=0$$

Here $a=k^2$, $b=2(k+1)$, $c=4$

$$\text{Disc.} = b^2-4ac$$
$$= [2(k+1)]^2-4(k^2)(4)$$
$$= 4(k^2+2k+1)-16k^2$$
$$= 4k^2+8k+4-16k^2$$
$$= -12k^2+8k+4=0$$

As the disc. of the given expression is a perfect square. Therefore the roots are rational and unequal.

So, $\text{Disc.}=0$

$$-12k^2+8k+4=0$$
$$-(12k^2-8k-4)=0$$
$$12k^2-8k-4=0$$
$$12k^2-12k+4k-4=0$$
$$12k(k-1)+4(k-1)=0$$
$$(12k+4)(k-1)=0$$

$12k+4=0$ OR $k-1=0$

$k=-\frac{4}{12}$ $\qquad$ $k=1$

$k=-1/3$

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Q. No. 4

Find the value of k, if the roots of the following equations are equal.

i $(2k-1)x^2 + 3kx + 3 = 0$

Here $a = 2k-1$, $b = 3k$, $c = 3$

As the roots are equal, so,

$$b^2 - 4ac = 0$$
$$(3k)^2 - 4(2k-1)(3) = 0$$
$$9k^2 - 12(2k-1) = 0$$
$$9k^2 - 24k + 12 = 0$$
$$3(3k^2 - 8k + 4) = 0$$
$$3k^2 - 8k + 4 = 0$$
$$3k^2 - 6k - 2k + 4 = 0$$
$$3k(k-2) - 2(k-2) = 0$$
$$(3k-2)(k-2) = 0$$

$3k-2 = 0$ OR $k-2 = 0$

$k = 2/3$ $\quad k = 2$

ii $x^2 + 2(k+2)x + 3k + 4 = 0$

Here $a = 1$, $b = 2(k+2)$, $c = 3k+4$

Disc. $= b^2 - 4ac$

As roots are equal, so.

Disc $= 0$

$$b^2 - 4ac = 0$$
$$[2(k+2)]^2 - 4(1)(3k+4) = 0$$
$$4(k^2 + 4k + 4) - 4(3k+4) = 0$$
$$4k^2 + 16k + 16 - 12k - 16 = 0$$
$$4k^2 + 4k = 0$$
$$4k(k+1) = 0$$

$4k = 0$ OR $k+1 = 0$

$k = 0$ $\quad k = -1$

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iii $(3k+2)x^2 - 5(k+1)x + (2k+3) = 0$

Here $a = 3k+2$, $b = -5(k+1)$, $c = 2k+3$

As the roots are equal,

so Disc. $= 0$

$b^2 - 4ac = 0$

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$[-5(k+1)]^2 - 4(3k+2)(2k+3) = 0$

$25(k^2+2k+1) - 4(6k^2+13k+6) = 0$

$25k^2+50k+25 - 24k^2 - 52k - 24 = 0$

$k^2 - 2k + 1 = 0$

$(k-1)^2 = 0$

$k - 1 = 0$

$k = 1$

Q.No.5

Show that the equation $x^2 + (mx+c)^2 = a^2$ has equal roots, if $c^2 = a^2(1+m^2)$

$x^2 + (mx+c)^2 = a^2$

$x^2 + m^2x^2 + 2mcx + c^2 = a^2$

$(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$

Here $a = 1+m^2$, $b = 2mc$, $c = c^2 - a^2$

As the roots are equal so,

Disc. $= 0$

$b^2 - 4ac = 0$

$(2mc)^2 - 4(1+m^2)(c^2-a^2) = 0$

$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - a^2m^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4mc^2 + 4a^2m^2 = 0$

$-4c^2 + 4a^2 + 4ac^2 + 4a^2m^2 = 0$

$-4(c^2 - a^2 - a^2m^2) = 0$

$c^2 = a^2 + a^2m^2$

$c^2 = a(1+m^2)$

Hence proved.

Q.No.6

Find the condition that the roots of the equation $(mx+c)^2-4ax=0$ are real and equal.

$$(mx+c)^2-4ax=0$$
$$m^2x^2+2mcx+c^2-4ax=0$$
$$m^2x^2+2(mc-2a)x+c^2=0$$

Here $a=m^2$, $b=2(mc-2a)$, $c=c^2$

As the roots are equal, so

$$\text{Disc.}=0$$
$$b^2-4ac=0$$
$$[2(mc-2a)]^2-4m^2c^2=0$$
$$4(m^2c^2-4amc+4a^2)-4m^2c^2=0$$
$$4(m^2c^2-4amc+4a^2-m^2c^2)=0$$
$$4(4a^2+4amc)=0$$
$$16(a^2+amc)=0$$
$$a^2+amc=0$$
$$a(a-mc)=0$$
$$a=mc$$

which is the required Equation.

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Q.No.7

If the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal, then $a=0$ or $a^3+b^3+c^3=3abc$.

$$(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$$

Here, $a=c^2-ab$, $b=-2(a^2-bc)$, $c=b^2-ac$

As the roots are equal.

$$\text{Disc.}=0$$
$$b^2-4ac=0$$

$$\left[-2(a^2-bc)\right]^2-4(c^2-ab)(b^2-ac)=0$$
$$4(a^4-2a^2bc+b^2c^2)-4(b^2c^2-ac^3-ab^3+a^2bc)=0$$
$$4(a^4-2a^2bc+b^2c^2-b^2c^2+ac^3+ab^3-a^2bc)=0$$
$$a^4+ab^3+ac^3-3a^2bc=0$$
$$a(a^3+b^3+c^3-3abc)=0$$
$a=0$ or $a^3+b^3+c^3-3abc=0$
$$a^3+b^3+c^3=3abc$$
Hence proved.

Q.No.8
Show that the roots of the following equations are rational.

i $\;a(b-c)x^2+b(c-a)x+c(a-b)=0$

$$a(b-c)x^2+b(c-a)x+c(a-b)=0$$
Here, $A=a(b-c)$, $B=b(c-a)$, $C=c(a-b)$
$$\text{Disc.}=B^2-4AC$$
$$=\left[b(c-a)\right]^2-4\left[a(b-c)\right]\left[c(a-b)\right]$$
$$=b^2(c^2+a^2-2ac)-4ac(ab-b^2-ac+bc)$$
$$=b^2c^2+a^2b^2-2ab^2c-4a^2bc+4ab^2c+4a^2c^2-4abc^2$$
$$=a^2b^2+b^2c^2+4a^2c^2+2ab^2c-4a^2bc-4abc^2$$
$$=(ab)^2+(bc)^2+(-2ac)^2+2(ab)(bc)+2(bc)(-2ac)+2(-2ac)(ab)$$
$$=(ab+bc-2ac)^2$$
Hence the roots are rational.

ii $\;(a+2b)x^2+2(a+b+c)x+(a+2c)=0$

Here, $A=a+2b$, $B=2(a+b+c)$, $C=a+2c$
$$\text{Disc.}=b^2-4ac$$
$$=\left[2(a+b+c)\right]^2-4(a+2b)(a+2c)$$
$$=4\left[(\cancel{a^2}+b^2+c^2+\cancel{2ab}+2bc+\cancel{2ca}-\cancel{a^2}-\cancel{2ac}-\cancel{2ab}-4bc)\right]$$
$$=4\left[b^2+c^2-2bc\right]$$
$$=4(b-c)^2$$
Hence the roots are rational.

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Q.No.9

For all values of k, prove that the roots of the equation

$$x^2-2\left(k+\frac{1}{k}\right)x+3=0 \quad k\neq 0 \text{ are real.}$$

$$x^2-2\left(k+\frac{1}{k}\right)x+3=0$$

Here $a=1$, $b=-2\left(k+\frac{1}{k}\right)$, $c=3$

$$\text{Disc.} = b^2-4ac$$

$$= \left[-2\left(k+\frac{1}{k}\right)\right]^2-4(1)(3)$$

$$= 4\left(k+\frac{1}{k}\right)^2-12$$

$$= 4\left[k^2+\frac{1}{k^2}+2-3\right]$$

$$= 4\left[k^2+\frac{1}{k^2}-1\right] > 0$$

Hence the roots are real.

Q.No.10

Show that the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are real.

$$(b-c)x^2+(c-a)x+(a-b)=0$$

Here $A=b-c$, $B=c-a$, $C=a-b$

$$\text{Disc.} = B^2-4AC$$

$$= (c-a)^2-4(b-c)(a-b)$$

$$= c^2+a^2-2ac-4(ab-b^2-ac+bc)$$

$$= a^2+c^2-2ac-4ab+4b^2+4ac-4bc$$

$$= a^2+4b^2+c^2-4ab-4bc+2ac$$

$$= a^2+(-2b)^2+(c)^2+2(a)(-2b)+2(-2b)(c)+2(a)(c)$$

$$= (a-2b+c)^2 > 0$$

Hence the roots of the equation are real.