Ex # 1.4

Solve the following equations.

Q.No.01 $2x+5=\sqrt{7x+16}$

$2x+5=\sqrt{7x+16}$ — (i)

Squaring both sides.

$(2x+5)^2=(\sqrt{7x+16})^2$

$4x^2+20x+25=7x+16$

$4x^2+20x+25-7x-16=0$

$4x^2+13x+9=0$

$4x^2+9x+4x+9=0$

$x(4x+9)+1(4x+9)=0$

$(x+1)(4x+9)=0$

Either $x+1=0$ or $4x+9=0$

$x=-1$ $\quad x=-9/4$

Check:

put $x=-1$ in eq(i)

$2(-1)+5=\sqrt{7(-1)+16}$

$-2+5=\sqrt{-7+16}$

$3=\sqrt{9}$

$3=3$

which is true

put $x=-9/4$ in eq.(i)

$2(-\frac{9}{4})+5=\sqrt{7(-\frac{9}{4})+16}$

$-\frac{9}{2}+5=\sqrt{-\frac{63}{4}+16}$

$\frac{-9+10}{2}=\sqrt{\frac{63+64}{4}}$

$\frac{1}{2}=\sqrt{\frac{1}{4}}$

$\frac{1}{2}=\frac{1}{2}$

which is true.

Thus, solution set $=\{-1, -\frac{9}{4}\}$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Q.No.2 $\sqrt{x+3} = 3x-1$

$\sqrt{x+3} = 3x-1$ ——— (i)

Squaring both sides.

$(\sqrt{x+3})^2 = (3x-1)^2$

$x+3 = 9x^2-6x+1$

$9x^2-6x-x+1-3=0$

$9x^2-7x-2=0$

$9x^2-9x+2x-2=0$

$9x(x-1)+2(x-1)=0$

$(9x+2)(x-1)=0$

Either $9x+2=0$ or $x-1=0$

$9x=-2$ $x=1$

$x=-2/9$

Check:

Put $x=\frac{-2}{9}$ in Eq.i

$\sqrt{\frac{-2}{9}+3} = 3(\frac{-2}{9})-1$

$\sqrt{\frac{-2+27}{9}} = \frac{-2}{3}-1$

$\sqrt{\frac{25}{9}} = \frac{-2-3}{3}$

$\frac{5}{3} \neq -\frac{5}{3}$

which is not true.

Put $x=1$ in eq.(i)

$\sqrt{1+3} = 3(1)-1$

$2 = 2$

which is true.

Thus solution set $= \{1\}$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Q.NO.3 $4x = \sqrt{13x+14} - 3$

$4x+3 = \sqrt{13x+14}$ ——— (i)

Squaring both sides

$(4x+3)^2 = (\sqrt{13x+14})^2$

$16x^2+24x+9 = 13x+14$

$16x^2+24x-13x+9-14=0$

$16x^2+11x-5=0$

$16x^2+16x-5x-5=0$

$16x(x+1)-5(x+1)=0$

$(16x-5)(x+1)=0$

Either $16x-5=0$ or $x+1=0$

$16x=5$ $x=-1$

$x=5/16$

Check:

Put $x=5/16$ in eq.(i)

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14}-3$

$\frac{5}{4} = \sqrt{\frac{65}{16}+14}-3$

$\frac{5}{4} = \sqrt{\frac{289}{16}}-3$

$\frac{5}{4} = \frac{17}{4}-3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$

Which is true

Put $x=-1$ in eq.(i)

$4(-1) = \sqrt{13(-1)+14}-3$

$-4 = \sqrt{-13+14}-3$

$-4 = \sqrt{1}-3$

$-4 = 1-3$

$-4 = -2$

which is not true.

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Thus solution set $= \left\{\frac{5}{16}\right\}$

Q.No.4 $\sqrt{3x+100} - x = 4$

$\sqrt{3x+100} = x+4$ — (i)

Squaring both sides

$(\sqrt{3x+100})^2 = (x+4)^2$

$3x+100 = x^2+16x+16$

$x^2+8x+16-3x-100=0$

$x^2+5x-84=0$

$x^2+12x-7x-84=0$

$x(x+12)-7(x+12)=0$

$(x-7)(x+12)=0$

Either $x-7=0$ or $x+12=0$

$x=7$ $x=-12$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Check:

Put $x=7$ in eq.(i)

$\sqrt{3(7)+100} = 7+4$

$\sqrt{21+100} = 11$

$\sqrt{121} = 11$

$11 = 11$

Which is true

Put $x=-12$ in eq.(i)

$\sqrt{3(-12)+100} = -12+4$

$\sqrt{-36+100} = -8$

$\sqrt{64} = -8$

$8 \neq -8$

Which is not true

The solution set $= \{7\}$

Q.No.5 $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$ — (i)

Squaring both sides

$(\sqrt{x+5} + \sqrt{x+21})^2 = (\sqrt{x+60})^2$

$(x+5)+(x+21)+2\sqrt{(x+5)(x+21)} = x+60$

$x+5+x+21+2\sqrt{x^2+26x+105} = x+60$

$2x+26+2\sqrt{x^2+26x+105} = x+60$

$2x+26-x-60 = -2\sqrt{x^2+26x+105}$

$x-34 = -2\sqrt{x^2+26x+105}$

Squaring on both sides

$(x-34)^2 = (-2\sqrt{x^2+26x+105})^2$

$$x^2-68x+1156=4(x^2+26x+105)$$
$$x^2-68x+1156=4x^2+104x+420$$
$$4x^2-x^2+104x+68x+420-1156=0$$
$$3x^2+172x-736=0$$
$$3x^2-12x+184x-736=0$$
$$3x(x-4)+184(x-4)=0$$
$$(3x+184)(x-4)=0$$

Either $3x+184=0$ or $x-4=0$

$3x=-184$, $x=4$

$x=-\frac{184}{3}$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Check:

Put $x=-\frac{184}{3}$ in eq.(i)

$$\sqrt{-\frac{184}{3}+5}+\sqrt{-\frac{184}{3}+21}=\sqrt{-\frac{184}{3}+60}$$
$$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}}$$

Which is not true.

Put $x=4$ in eq.(i)

$$\sqrt{4+1}+\sqrt{4-2}=\sqrt{4+6}$$
$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$
$$3+5=8$$
$$8=8$$

which is true

Thus solution set $=\{4\}$

Q.No.06 $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6} \quad \text{---(i)}$$

Squaring on both sides

$$(\sqrt{x+1}+\sqrt{x-2})^2=(\sqrt{x+6})^2$$
$$(\sqrt{x+1})^2+(\sqrt{x-2})^2+2\sqrt{(x+1)(x-2)}=x+6$$
$$x+1+x-2+2\sqrt{(x+1)(x-2)}=x+6$$
$$2x-1+2\sqrt{x^2-x-2}=x+6$$
$$2x-x-1-6=(-2\sqrt{x^2-x-2})^2$$
$$x-7=-2\sqrt{x^2-x-2}$$

Squaring on both sides

$$(x-7)^2=(-2\sqrt{x^2-x-2})^2$$

$$x^2-14x+49=4(x^2-x-2)$$
$$x^2-14x+49=4x^2-4x-8$$
$$4x^2-x^2-4x+14x-8-49=0$$
$$3x^2+10x-57=0$$
$$3x^2-9x+19x-57=0$$
$$3x(x-3)+19(x-3)=0$$
$$(3x+19)(x-3)=0$$

Either $3x+19=0$ or $x-3=0$

$3x=-19$ $\quad x=3$

$x=-19/3$

Check:

put $x=-19/3$ in eq.(i)

$$\sqrt{-\frac{19}{3}+1}+\sqrt{-\frac{19}{3}-2}=\sqrt{-\frac{19}{3}+6}$$
$$\sqrt{-\frac{16}{3}}+\sqrt{-\frac{25}{3}}=\sqrt{-\frac{1}{3}}$$

which is not true.

put $x=3$ in eq.(i)

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$
$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$
$$2+1=3$$
$$3=3$$

which is true.

Thus solution set $=\{3\}$

Q.No.7 $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x} \quad \text{— (i)}$$

Squaring both sides.

$$(\sqrt{11-x}+\sqrt{6-x})^2=(\sqrt{27-x})^2$$
$$(\sqrt{11-x})^2+(\sqrt{6-x})^2+2\sqrt{(11-x)(6-x)}=27-x$$
$$11-x+6-x+2\sqrt{x^2-17x+66}=27-x$$
$$17-2x+2\sqrt{x^2-17x+66}=27-x$$
$$2\sqrt{x^2-17x+66}=27-17-x+2x$$
$$2\sqrt{x^2-17x+66}=10+x$$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Squaring both sides

$$(2\sqrt{x^2-17x+66})^2 = (10+x)^2$$

$$4(x^2-17x+66) = 100+x^2+20x$$

$$4x^2-68x+264-x^2-20x-100=0$$

$$3x^2-88x+164=0$$

$$3x^2-6x-82x+164=0$$

$$3x(x-2)-82(x-2)=0$$

$$(3x-82)(x-2)=0$$

Either $3x-82=0$ or $x-2=0$

$x=\frac{82}{3}$ $x=2$

Check:

Put $x=\frac{82}{3}$ in Eq(i)

$$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$

$$\sqrt{-\frac{49}{3}}+\sqrt{-\frac{64}{3}}=-\frac{1}{3}$$

Which is not true

Put $x=2$ in eq(i)

$$\sqrt{11-2}+\sqrt{6-2}=\sqrt{27-2}$$

$$\sqrt{9}+\sqrt{4}=\sqrt{25}$$

$$3+2=5$$

$$5=5$$

which is true.

Thus solution set $=\{2\}$.

Q.No. 8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a} \quad \text{---(i)}$$

Squaring both sides

$$(\sqrt{4a+x}-\sqrt{a-x})^2=(\sqrt{a})^2$$

$$(\sqrt{4a+x})^2+(\sqrt{a-x})^2-2\sqrt{(4a+x)(a-x)}=a$$

$$4a+x+a-x-2\sqrt{(4a+x)(a-x)}=a$$

$$5a-2\sqrt{(4a+x)(a-x)}=a$$

$$5a-a=2\sqrt{4a^2-3ax-x^2}$$

$$4a=2\sqrt{4a^2-3ax-x^2}$$

Squaring on both sides

$$16a^2=4(4a^2-3ax-x^2)$$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

$$16a^2 = 16a^2 - 12ax - 4x^2$$

$$16a^2 - 16a^2 + 12ax + 4x^2 = 0$$

$$4x(x + 3a) = 0$$

Either $4x = 0$ $\qquad$ $x + 3a = 0$

$x = 0$ $\qquad$ $x = -3a$

Check

put $x = 0$ in Eq(i)

$$\sqrt{4a+0} - \sqrt{a-0} = \sqrt{a}$$

$$\sqrt{4a} - \sqrt{a} = \sqrt{a}$$

$$2\sqrt{a} - \sqrt{a} = \sqrt{a}$$

$$\sqrt{a} = \sqrt{a}$$

which is true

put $x = -3a$ in Eq(i)

$$\sqrt{4a-3a} - \sqrt{a+3a} = \sqrt{a}$$

$$\sqrt{a} - \sqrt{4a} = \sqrt{a}$$

$$\sqrt{a} - 2\sqrt{a} = \sqrt{a}$$

$$-\sqrt{a} = \sqrt{a}$$

which is not true

Thus solution set $= \{0\}$

Q.No.9 $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

$$\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1 \longrightarrow (i)$$

Let $x^2 + x = y$,

Then eq.(i) becomes.

$$\sqrt{y+1} - \sqrt{y-1} = 1$$

Squaring on both sides:-

$$(\sqrt{y+1} - \sqrt{y-1})^2 = (1)^2$$

$$(\sqrt{y+1})^2 + (\sqrt{y-1})^2 - 2\sqrt{(y+1)(y-1)} = 1$$

$$y + 1 + y - 1 - 2\sqrt{y^2-1} = 1$$

$$2y - 1 = 2\sqrt{y^2-1}$$

Squaring both sides:-

$$(2y-1)^2 = (2\sqrt{y^2-1})^2$$

$$4y^2 + 1 - 4y = 4(y^2-1)$$

$$4y^2 + 1 - 4y = 4y^2 - 4$$

$$4y^2 + 1 - 4y - 4y^2 + 4 = 0$$

$$-4y + 5 = 0$$

$$-4y = -5 \Rightarrow y = 5/4$$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

put $y = \frac{5}{4}$ in $x^2 + x = y$

$$x^2 + x = \frac{5}{4}$$

$$4x^2 + 4x = 5$$

$$4x^2 + 4x - 5 = 0$$

By using quadratic formula:-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$= \frac{-4 \pm \sqrt{16 + 80}}{8}$$

$$= \frac{-4 \pm \sqrt{96}}{8} \Rightarrow \frac{-4 \pm 4\sqrt{6}}{8}$$

$$= \frac{4(-1 \pm \sqrt{6})}{8} \Rightarrow \frac{-1 \pm \sqrt{6}}{2}$$

$$x = \frac{-1 \pm \sqrt{6}}{2}$$

Q.No.10 $\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3$

$$\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3 \quad \text{— (i)}$$

Let $x^2 + 3x = y$

so $\sqrt{y+8} + \sqrt{y+2} = 3$

Squaring on both sides.

$$(\sqrt{y+8} + \sqrt{y+2})^2 = (3)^2$$

$$(\sqrt{y+8})^2 + (\sqrt{y+2})^2 + 2\sqrt{(y+8)(y+2)} = 9$$

$$y + 8 + y + 2 - 9 = -2\sqrt{(y+8)(y+2)}$$

$$2y + 1 = 2\sqrt{y^2 + 10y + 16}$$

Squaring on both sides:

$$(2y+1)^2 = (2\sqrt{y^2+10y+16})^2$$

$$4y^2 + 1 + 4y = 4(y^2 + 10y + 16)$$

$$4y^2+1+4y = 4y^2+40y+64$$

$$4y^2-4y^2+4y-40y+1-64=0$$

$$-(36y+63)=0$$

$$36y+63=0$$

$$36y=-63$$

$$y=\frac{-63}{36}$$

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Put $y=\frac{-63}{36}$ in $x^2+3x=y$

$$x^2+3x=-\frac{63}{36}$$

$$36x^2+108x=-63$$

$$36+108x+63=0$$

By using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$=\frac{-108\pm\sqrt{11664-9072}}{72}$$

$$=\frac{-108\pm\sqrt{2592}}{72}$$

$$=\frac{36(-3\pm\sqrt{2})}{72}$$

$$x=\frac{-3\pm2}{2}$$

Thus solution set $=\left\{\frac{-3\pm2}{2}\right\}$.

Q.No.11 $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Let $x^2+3x=y$

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring on both sides

$$(\sqrt{y+9}+\sqrt{y+4})^2=(5)^2$$

$$(\sqrt{y+9})^2+(\sqrt{y+4})^2+2\sqrt{(y+9)(y+4)}=25$$

$$y+9+y+4+2\sqrt{y^2+13y+36}=25$$

$$2y+13+2\sqrt{y^2+13y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=12-2y$$

Squaring both sides

$$(2\sqrt{y^2+13y+36})^2=(12-2y)^2$$

$$4(y^2+13y+36)=4(36+y^2-12y)$$

$$y^2+13y+36=y^2-12y+36$$

$$y^2-y^2+13y+12y+36-36=0$$

$$25y=0$$

$$y=0$$

As $x^2+3x=y$

So $x^2+3x=0$

$x(x+3)=0$

Either $x=0$ or $x+3=0$

$x=-3$

Thus solution set $=\{-3,0\}$.

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abdul Qadeer Malik
PhD(Mathematics Scholar)
HOD Math Department
0341-5838491