Ex#1.3

Solve the following Equations.

Q.No.1 $2x^4-11x^2+5=0$

$$2x^4-11x^2+5=0 \quad \text{---} \quad (i)$$

Let $x^2=y$ then $x^4=y^2$

So Eq. (i) becomes

$$2y^2-11y+5=0$$
$$2y^2-10y-y+5=0$$
$$2y(y-5)-1(y-5)=0$$
$$(2y-1)(y-5)=0$$

Either $2y-1=0$ or $y-5=0$

$y=\frac{1}{2}$ $\quad$ $y=5$

Since $x^2=y$

$x^2=\frac{1}{2}$ $\quad$ $x^2=5$

$\sqrt{x^2}=\pm\sqrt{\frac{1}{2}}$ $\quad$ $\sqrt{x^2}=\pm\sqrt{5}$

$x=\pm\frac{1}{\sqrt{2}}$ $\quad$ $x=\pm\sqrt{5}$

Thus solution set $=\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Q.No.2 $2x^4=9x^2-4$

$$2x^4-9x^2+4=0 \quad \text{---} \quad (i)$$

Let $x^2=y$ then $x^4=y^2$

So Eq. (i) becomes.

$$2y^2-9y+4=0$$
$$2y^2-8y-y+4=0$$
$$2y(y-4)-1(y-4)=0$$
$$(2y-1)(y-4)=0$$

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Either $2y-1=0$ or $y-4=0$

$y=\frac{1}{2}$ $\quad$ $y=4$

put $y=\frac{1}{2}, 4$ in $x^2=y$, we get.

$x^2=\frac{1}{2}$ $\quad$ $x^2=4$

$\sqrt{x^2}=\pm\sqrt{\frac{1}{2}}$ $\quad$ $\sqrt{x^2}=\pm\sqrt{4}$

$x=\pm\frac{1}{\sqrt{2}}$ $\quad$ $x=\pm 2$

Thus solution set $=\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

Q.No.3 $5x^{1/2}=7x^{1/4}-2$

$5x^{1/2}-7x^{1/4}+2=0$ — (i)

Let $x^{1/4}=y$ then $x^{1/2}=y^2$

So Eq. (i) becomes

$5y^2-7y+2=0$

$5y^2-5y-2y+2=0$

$5y(y-1)-2(y-1)=0$

$(5y-2)(y-1)=0$

Either $5y-2=0$ or $y-1=0$

$y=\frac{2}{5}$ $\quad$ $y=1$

$\therefore$ $x^{1/4}=y$

$x^{1/4}=\frac{2}{5}$ $\quad$ $x^{1/4}=1$

$(x^{1/4})^4=(\frac{2}{5})^4$ $\quad$ $(x^{1/4})^4=1$

$x=\frac{16}{625}$ $\quad$ $x=1$

Thus solution set $=\left\{\frac{16}{626}, 1\right\}$

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Q.No.4

$$x^{2/3} + 54 = 15x^{1/3}$$

$$x^{2/3} - 15x^{1/3} + 54 = 0 \quad \text{— (i)}$$

Let $x^{1/3} = y$ then $x^{2/3} = y$

So Eq. (i) becomes

$$y^2 - 15y + 54 = 0$$

$$y^2 - 9y - 6y + 54 = 0$$

$$y(y-9) - 6(y-9) = 0$$

$$(y-6)(y-9) = 0$$

$y - 6 = 0$ $\qquad$ $y - 9 = 0$

$y = 6$ $\qquad$ $y = 9$

$\therefore \quad x^{1/3} = y$

$x^{1/3} = 6$, $\qquad$ $x^{1/3} = 9$

$(x^{1/3})^3 = (6)^3$, $\qquad$ $(x^{1/3})^3 = (9)^3$

$x = 216$, $\qquad$ $x = 729$

Thus solution set $= \{729, 216\}$

Q.No.5

$$3x^{-2} + 5 = 8x^{-1}$$

$$3x^{-2} - 8x^{-1} + 5 = 0 \quad \text{— (i)}$$

Let $x^{-1} = y$ then $x^{-2} = y^2$

Eq. (i) becomes

$$3y^2 - 8y + 5 = 0$$

$$3y^2 - 5y - 3y + 5 = 0$$

$$y(3y-5) - 1(3y-5) = 0$$

$$(y-1)(3y-5) = 0$$

Either $y - 1 = 0$ $\qquad$ OR. $3y - 5 = 0$

$y = 1$ $\qquad$ $y = 5/3$

$\therefore \quad x^{-1} = y$

$\frac{1}{x} = 1$ $\qquad$ $\frac{1}{x} = \frac{5}{3}$

$x = 1$ $\qquad$ $x = \frac{3}{5}$

Thus solution set $= \{1, 3/5\}$

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Q.No.6 $(2x^2+1)+\frac{3}{2x^2+1}=4$

$(2x^2+1)+\frac{3}{2x^2+1}=4 \longrightarrow (i)$

Let $2x^2+1=y$

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So, Eq. (i) becomes:

$y+\frac{3}{y}=4$

Multiplying both sides by "y".

$y^2+3=4y$

$y^2-4y+3=0$

$y^2-3y-y+3=0$

$y(y-3)-1(y-3)=0$

$(y-1)(y-3)=0$

Either $y-1=0$ or $y-3=0$

$y=1$ $\quad$ $y=3$

$\therefore \quad 2x^2+1=y$

$2x^2+1=1$

$2x^2=1-1$

$2x^2=0$

$x^2=0$

$x=0$

$2x^2+1=3$

$2x^2=3-1$

$2x^2=2$

$x^2=1$

$x=\pm 1$

Thus solution set $=\{-1,0,1\}$

Q.No.7 $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4 \quad -(i)$

Let $\frac{x}{x-3}=y$

So Eq. (i) becomes

$y+4\left(\frac{1}{y}\right)=4$

Multiplying both sides by "y"

$y^2+4=4y$

$$y^2 - 4y + 4 = 0$$

$$y^2 - 2(y)(2) + (2)^2 = 0$$

$$(y-2)^2 = 0$$

$$y - 2 = 0$$

$$y = 2$$

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$$\therefore \quad \frac{x}{x-3} = y$$

$$\frac{x}{x-3} = 2$$

$$x = 2(x-3)$$

$$x = 2x - 6$$

$$2x - x = 6$$

$$x = 6$$

Thus solution set $= \{6\}$

Q.No.8 $\dfrac{4x+1}{4x-1} + \dfrac{4x-1}{4x+1} = 2\dfrac{1}{6}$

$$\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = \frac{13}{6} \quad \text{— (i)}$$

Let $\dfrac{4x+1}{4x-1} = y$

So Eq. (i) becomes.

$$y + \frac{1}{y} = \frac{13}{6}$$

Multiplying both sides by $6y$

$$6y^2 + 6 = 13y$$

$$6y^2 - 13y + 6 = 0$$

$$6y^2 - 9y - 4y + 6 = 0$$

$$3y(2y-3) - 2(2y-3) = 0$$

$$(3y-2)(2y-3) = 0$$

Either $3y - 2 = 0$ or $2y - 3 = 0$

$$y = \frac{2}{3} \qquad y = \frac{3}{2}$$

$$\therefore \quad \frac{4x+1}{4x-1} = y$$

$\frac{4x+1}{4x-1} = \frac{2}{3}$ $\qquad$ $\frac{4x+1}{4x-1} = \frac{3}{2}$

$3(4x+1) = 2(4x-1)$ $\qquad$ $2(4x+1) = 3(4x-1)$

$12x + 3 = 8x - 2$ $\qquad$ $8x + 2 = 12x - 3$

$12x - 8x = -2 - 3$ $\qquad$ $8x - 12x = -3 - 2$

$4x = -5$ $\qquad$ $-4x = -5$

$x = \frac{-5}{4}$ $\qquad$ $x = \frac{5}{4}$

Thus solution set $= \{ \pm 5/4 \}$

Q.No.9 $\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$

$\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$ ——— (i)

Let $\frac{x-a}{x+a} = y$

So Eq. (i) becomes

$y - \frac{1}{y} = \frac{7}{12}$

Multiplying both sides by "12y"

$12y^2 - 12 = 7y$

$12y^2 - 7y - 12 = 0$

$12y^2 - 16y + 9y - 12 = 0$

$4y(3y-4) + 3(3y-4) = 0$

$(4y+3)(3y-4) = 0$

Either $4y + 3 = 0$ or $3y - 4 = 0$

$4y = -3$ $\qquad$ $3y = 4$

$y = -3/4$ $\qquad$ $y = 4/3$

$\therefore \frac{x-a}{x+a} = y$

$\frac{x-a}{x+a} = -\frac{3}{4}$, $\frac{x-a}{x+a} = \frac{4}{3}$

$4(x-a) = -3(x+a)$, $3(x-a) = 4(x+a)$

$4x - 4a = -3x - 3a$, $3x - 3a = 4x + 4a$

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$4x+3x=4a-3a$

$7x=a$

$x=\frac{a}{7}$

$4x-3x=-4a-3a$

$x=-7a$

Thus solution set $=\left\{\frac{a}{7}, -7a\right\}$

Q.No.10

$$x^4-2x^3-2x^2+2x+1=0$$

$x^4-2x^3-2x^2+2x+1=0$

Dividing each term by x^2

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \rightarrow (i)$$

Let $x-\frac{1}{x}=y$

Squaring:-

$$\left(x-\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}-2=y^2$$

$$x^2+\frac{1}{x^2}=y^2+2$$

So Eq.(i) becomes

$$y^2+2-2y-2=0$$

$$y^2-2y=0$$

$$y(y-2)=0$$

Either $y=0$ or $y-2=0$

$y=2$

$\therefore \quad x-\frac{1}{x}=y$

$x-\frac{1}{x}=0$,

$x^2-1=0$,

$x^2=1$,

$x=\pm 1$

$x-\frac{1}{x}=2$

$x^2-1=2x$

$x^2-2x-1=0$

By using quadratic formula.

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$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} \Rightarrow \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(1 \pm \sqrt{2})}{2}$$

$$= 1 \pm \sqrt{2}$$

Thus solution set $= \{\pm 1, 1 \pm \sqrt{2}\}$

Q.No.11

$$2x^4 + x^3 - 6x^2 + x + 2 = 0$$

Dividing both sides by x^2

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \text{---(i)}$$

Let $\left(x + \frac{1}{x}\right) = y$

Squaring:-

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So, Eq.(i) becomes.

$$2(y^2 - 2) + y - 6 = 0$$

$$2y^2 - 4 + y - 6 = 0$$

$$2y^2 + y - 10 = 0$$

$$2y^2 + 5y - 4y - 10 = 0$$

$$y(2y + 5) - 2(2y + 5) = 0$$

$$(y - 2)(2y + 5) = 0$$

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$y-2=0$ | $2y+5=0$

$y=2$ | $y=-5/2$

$\therefore x+\frac{1}{x}=y$

$x+\frac{1}{x}=2$ | $x+\frac{1}{x}=-\frac{5}{2}$

$x^2+1=2x$ | $x^2+1=-\frac{5x}{2}$

$x^2-2x+1=0$ | $2x^2+2=-5x$

$(x-1)^2=0$ | $2x^2+5x+2=0$

$x-1=0$ | $2x^2+4x+x+2=0$

$x=1$ | $2x(x+2)+1(x+2)=0$

$(2x+1)(x+2)=0$

Either $2x+1=0$ or $x+2=0$

$x=\frac{-1}{2}$, $x=-2$

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Thus solution set $=\{1,-2,-\frac{1}{2}\}$

Que.12 $4.2^{2x+1}-9.2^x+1=0$

$4\cdot 2^{2x+1}-9\cdot 2^x+1=0$

$4\cdot 2^{2x}\cdot 2-9\cdot 2^x+1=0$ — (i)

Let $2^x=y$ then $2^{2x}=y^2$

So Eq.(i) becomes:-

$4y^2\cdot 2-9y+1=0$

$8y^2-9y+1=0$

$8y^2-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(8y-1)(y-1)=0$

Either $8y-1=0$ or $y-1=0$

$y=1/8$ | $y=1$

$\therefore 2^x=y$

$2^x=\frac{1}{8}$ | $2^x=1$

$2^x=1/2^3$ | $2^x=2^0$

$x=0$

$2^x = 2^{-3}$

$x = -3$

Thus solution set $= \{-3, 0\}$

Que #13 $3^{2x+2} = 12 \cdot 3^x - 3$

$3^{2x+2} = 12 \cdot 3^x - 3$

$3^{2x} \cdot 3^2 - 12 \cdot 3^x + 3 = 0$

$9 \cdot 3^{2x} - 12 \cdot 3^x + 3 = 0$ — (i)

Let $3^x = y$ then $3^{2x} = y^2$

So, Eq. (i) becomes

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(9y-3)(y-1) = 0$

Either $9y - 3 = 0$ or $y - 1 = 0$

$9y = 3$ $\quad y = 1$

$y = 3/9$

$y = \frac{1}{3}$

$\because 3^x = y$

$3^x = \frac{1}{3}$ $\quad 3^x = 1$

$3^x = 3^{-1}$ $\quad 3^x = 3^0$

$x = -1$ $\quad x = 0$

Thus solution set $= \{-1, 0\}$

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Q.No 14

$$2^x + 64 \cdot 2^{-x} - 20 = 0$$

$2^x + 64 \cdot 2^{-x} - 20 = 0$ — (i)

Let $2^x = y$. then $2^{-x} = \frac{1}{y}$

So Eq.(i) becomes

$y + 64 \cdot \frac{1}{y} - 20 = 0$

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$

$y^2 - 16y - 4y + 64 = 0$

$y(y-16) - 4(y-16) = 0$

$(y-4)(y-16) = 0$

Either $y - 4 = 0$ or $y - 16 = 0$

$y = 4$ $y = 16$

$\therefore 2^x = y$

$2^x = 4$ $2^x = 16$

$2^x = 2^2$ $2^x = 2^4$

$x = 2$ $x = 4$

Thus the solution set $= \{2, 4\}$

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Q.No.15

$$(x+1)(x+3)(x-5)(x-7) = 192$$

$(x+1)(x+3)(x-5)(x-7) = 192$

As $1 - 5 = 3 - 7$

So, $[(x+1)(x-5)][(x+3)(x-7)] = 192$

$[x^2 - 5x + x - 5][x^2 - 7x + 3x - 21] = 192$

$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192$ — (i)

Let $x^2 - 4x = y$

so Eq.(i) becomes

$(y - 5)(y + 21) = 192$

$y^2 - 21y - 5y + 105 = 192$

$y^2 - 26y + 105 - 192 = 0$

$y^2 - 26y - 87 = 0$

$y^2 - 29y + 3y - 87 = 0$

$y(y-29) + 3(y-29) = 0$

$(y+3)(y-29) = 0$

Either $y+3=0$ or $y-29=0$

$y = -3$ $\quad$ $y = 29$

As $x^2 - 4x = y$

$x^2 - 4x = -3$

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$

$(x-1)(x-3) = 0$

Either $x-1=0$ or $x-3=0$

$x = 1$ $\quad$ $x = 3$

$x^2 - 4x = 29$

$x^2 - 4x - 29 = 0$

By using quadratic formula:-

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16+116}}{2}$$

$$x = \frac{4 \pm \sqrt{132}}{2}$$

$$x = \frac{4 \pm 2\sqrt{33}}{2}$$

$$x = 2\left(\frac{2 \pm \sqrt{33}}{2}\right)$$

$x = 2 \pm \sqrt{33}$

Thus solution set $= \{1, 3, 2 \pm \sqrt{33}\}$

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Q.No.16

$$(x-1)(x-2)(x-8)(x+5)+360=0$$

$(x-1)(x-2)(x-8)(x+5)+360=0$

As $-1-2 = -8+5$

$-3 = -3$

So, $[(x-1)(x-2)][(x-8)(x+5)]+360=0$

$[x^2-2x-x+2][x^2+5x-8x-40]+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0$ — (i)

Let $x^2 - 3x = y$

So Eq. (1) becomes.

$$(y+2)(y-40)+360=0$$
$$y^2-40y+2y-80+360=0$$
$$y^2-38y+280=0$$
$$y^2-28y-10y+280=0$$
$$y(y-28)-10(y-28)=0$$
$$(y-10)(y-28)=0$$

Either $y-10=0$ or $y-28=0$

$y=10$ $\quad$ $y=28$

$\because\ x^2-3x=y$

$x^2-3x=10$
$x^2-3x-10=0$
$x^2-5x+2x-10=0$
$x(x-5)+2(x-5)=0$
$(x+2)(x-5)=0$

Either $x+2=0$ or $x-5=0$

$x=-2$ $\quad$ $x=5$

$x^2-3x=28$
$x^2-3x-28=0$
$x^2-7x+4x-28=0$
$x(x-7)+4(x-7)=0$
$(x+4)(x-7)=0$

Either $x+4=0$ or $x-7=0$

$x=-4$ $\quad$ $x=7$

Thus solution set $=\{-4,-2,5,7\}$

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